Thermo-acoustic Oscillation of in a Closed Tube by Numerical Simulation

[2] Information Technology Center,
Nagoya University

Mashiro Ishigaki[1], Katsuya Ishii[1,2], Sizuko Adachi[3]

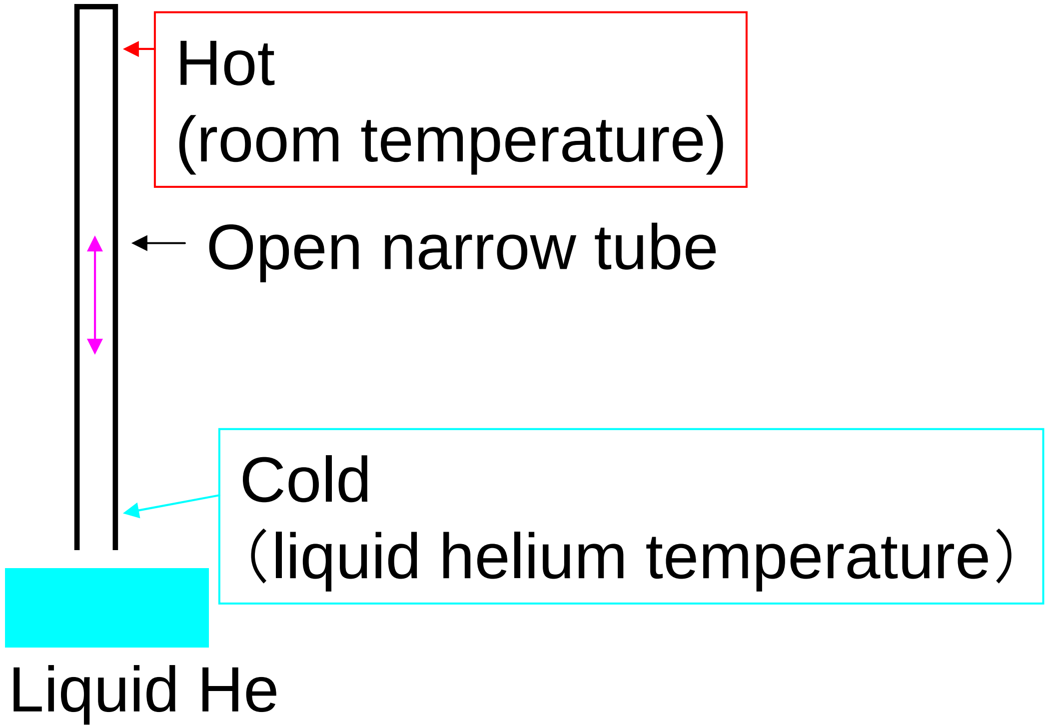
[1] Department of Computational Science and Engineering, Graduate School of Engineering, Nagoya University

[2] Information Technology Center, Nagoya University

[3] Tokyo International University

1.1 Taconis oscillation

Taconis oscillation is one of the thermo-acoustic oscillations.



Experimental study
- Taconis et al (1949)

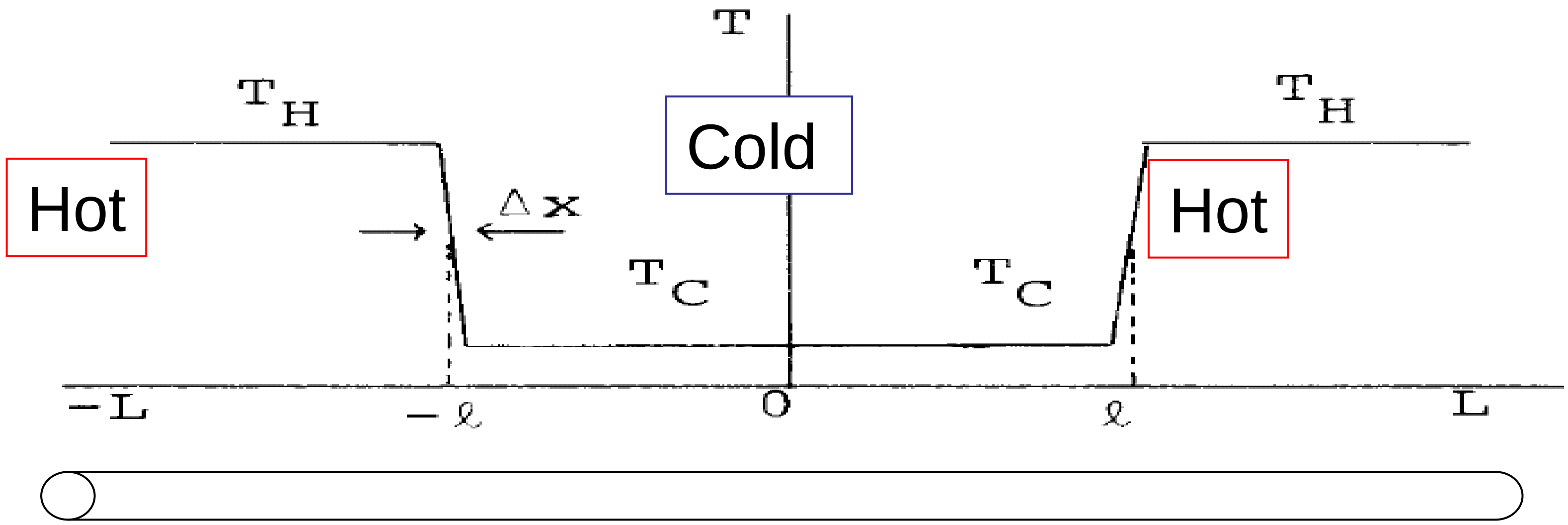
Theoretical study
- Kramers (1949)
- Rott (1969,1973)
- Sugimoto et al (2006-2008)

The tube has large temperature gradient axially.

The spontaneous oscillation is observed in the tube.

1.2 Experiment by Yazaki et al

(J.Low.Temp.Phys., **41**, 1980)



Closed tube

Yazaki et al observed a standing wave for different large temperature ratios T_H/T_C.
They explained these phenomena by using the theory of Taconis oscillation in a open-closed tube.

1.3 Thermo-acoustic Engine

G.B. Chen, T. Jin (Cryogenics, **39,** 1999, 843-846)

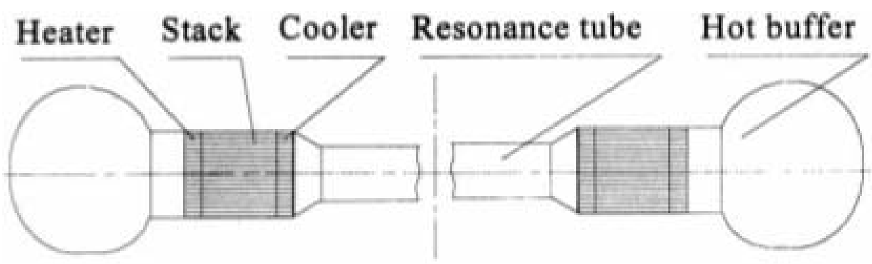


Fig. 1. Schematic of thermoacoustic engine.

The Pressure amplitude is measured when the heating temperature is changed.

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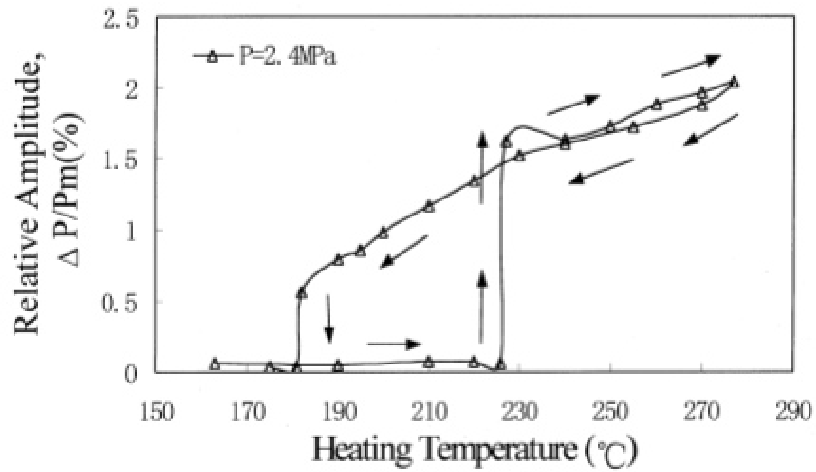
The hysteresis phenomenon is observed.

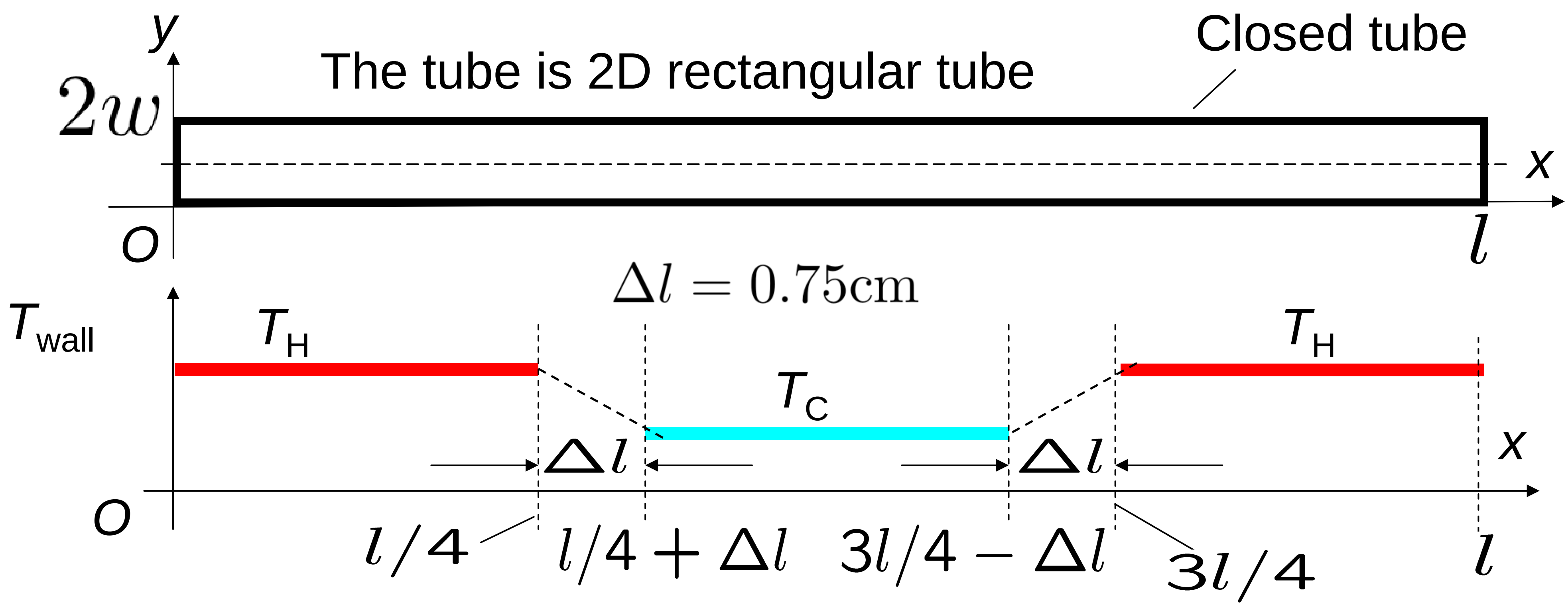


1.4 Objective

We observed hysteresis phenomena in Taconis oscillation which has not been reported experimentally.

The objectives are to analyze the Taconis oscillation in a closed tube using the numerical simulations, and to investigate the mechanism of hysteresis phenomenon.

2.1 Geometry



The fluid in the tube is gaseous helium at the room temperature T_H=300 K and the pressure p_0 =0.175x10^5 Pa, 1x10^5 Pa.
(We use different initial pressures p_0 in order to obtain the different thickness of the boundary layers.)

2.2 Basic equations

2D compressible Navier-Stokes equations

$$\partial_t \boldsymbol{q} + \partial_x \boldsymbol{E} + \partial_y \boldsymbol{F} = 1/Re(\partial_x \boldsymbol{R} + \partial_y \boldsymbol{S})$$

$$\boldsymbol{q} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \end{pmatrix}, \quad \boldsymbol{E} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (e+p)u \end{pmatrix}, \quad \boldsymbol{F} = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (e+p)v \end{pmatrix}$$

$$\boldsymbol{R} = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ R_4 \end{pmatrix}, \boldsymbol{S} = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ S_4 \end{pmatrix}$$

$$\begin{aligned} \tau_{xx} &= 4/3\mu u_x - 2/3\mu v_y \\ \tau_{xy} &= \mu(u_y + v_x) \\ \tau_{yy} &= 4/3\mu v_y - 2/3\mu u_x \\ R_4 &= u\tau_{xx} + v\tau_{xy} + \alpha\partial_x a^2 \\ S_4 &= u\tau_{xy} + v\tau_{yy} + \alpha\partial_y a^2 \\ a^2 &= \gamma(\gamma-1)[e/\rho - 1/2(u^2+v^2)] \\ \alpha &= \frac{k}{Pr(\gamma-1)} \end{aligned}$$

ρ: density
u, v: velocity
e: total energy density
p: pressure
a: acoustic velocity
μ: viscosity
k : thermal conductivity
Pr: Prandtl number
γ: specific heat ratio

Stokes hypothesis
Sutherland law

2.3 Numerical calculation

- The block pentadiagonal matrix scheme

  time development: 2^{nd}-order accurate three-point backward scheme

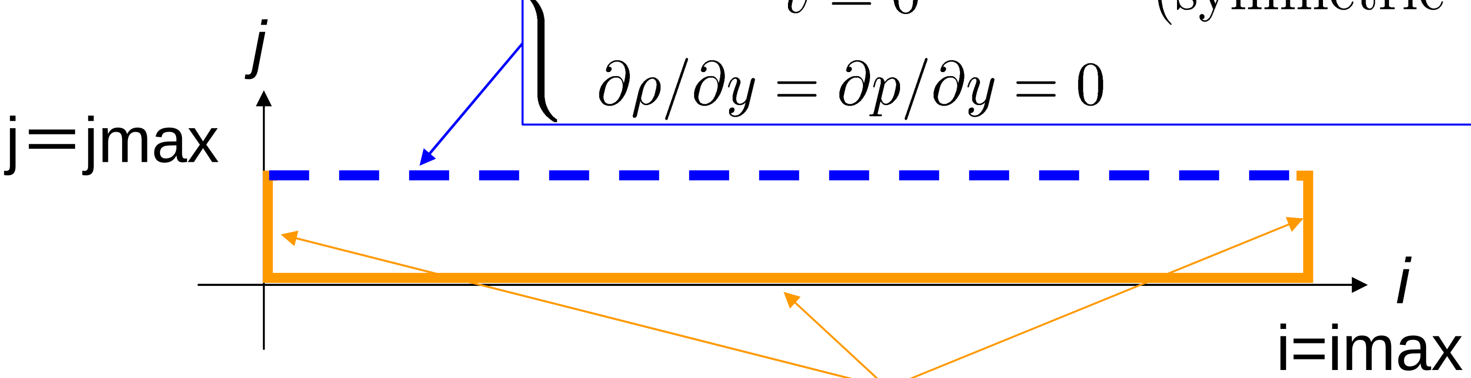
  convective terms: 4^{th} -order accurate central differencing

  viscous terms: 2^{nd} -order accurate central differencing

- Grid 300 x 36

- Boundary conditions

$$\begin{cases} \partial u / \partial y = 0 \\ v = 0 \qquad \text{(symmetric boundary)} \\ \partial \rho / \partial y = \partial p / \partial y = 0 \end{cases}$$



$$\begin{cases} u = v = 0 \\ \partial p / \partial n = 0 \quad \text{(tube wall)} \\ T = T_{\text{wall}} \end{cases}$$

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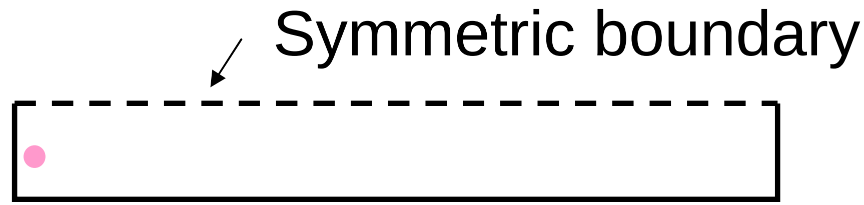
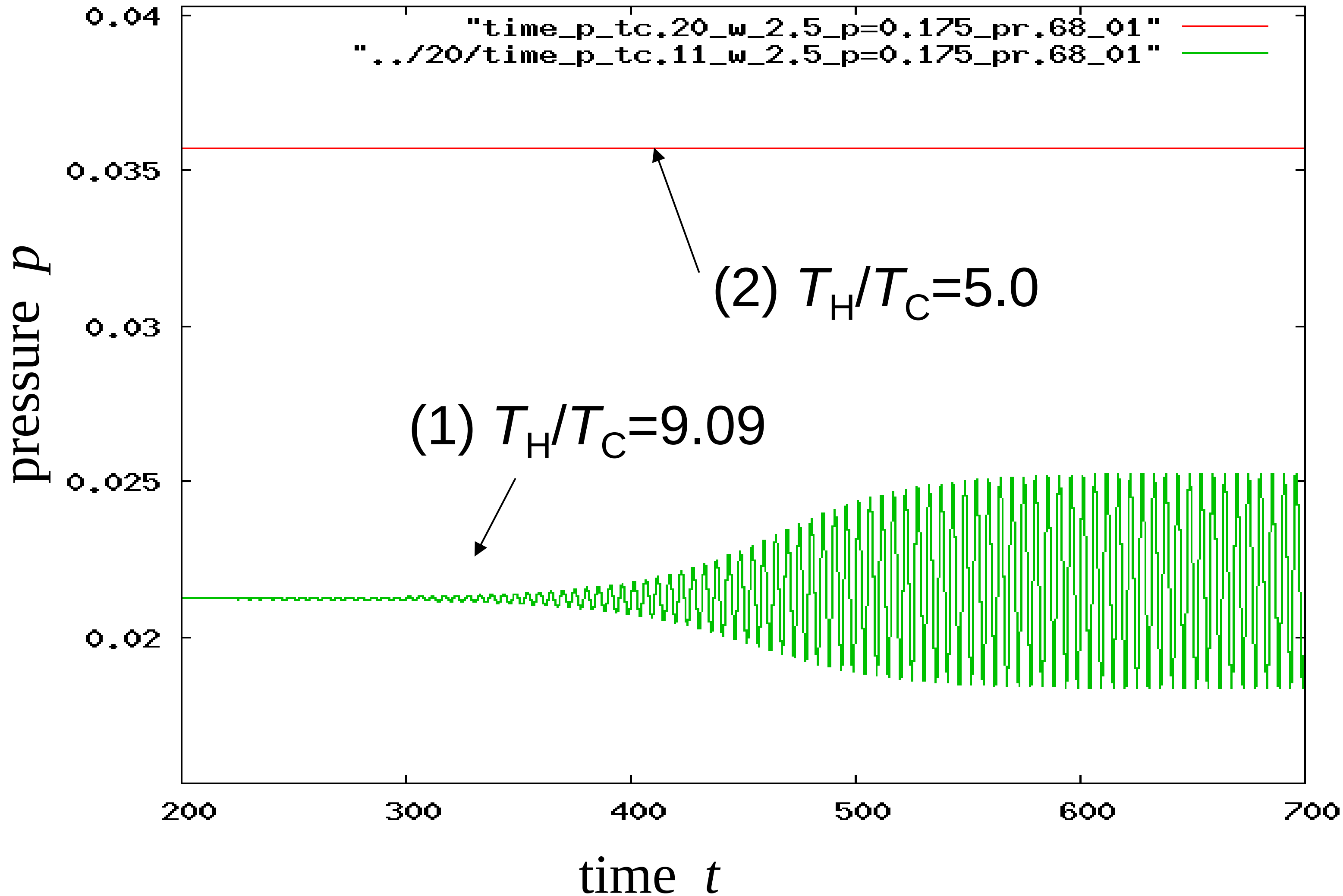
(1) Steady oscillation state

(2) Quiescent state



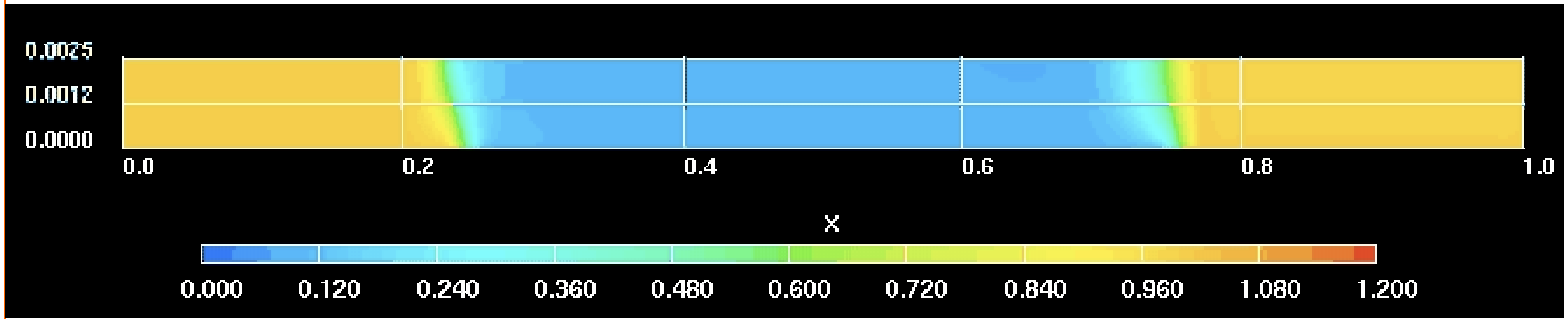
p_0=0.175x10^5 Pa



3.2 Thickness of thermal boundary layer

T_H/T_C=9.09

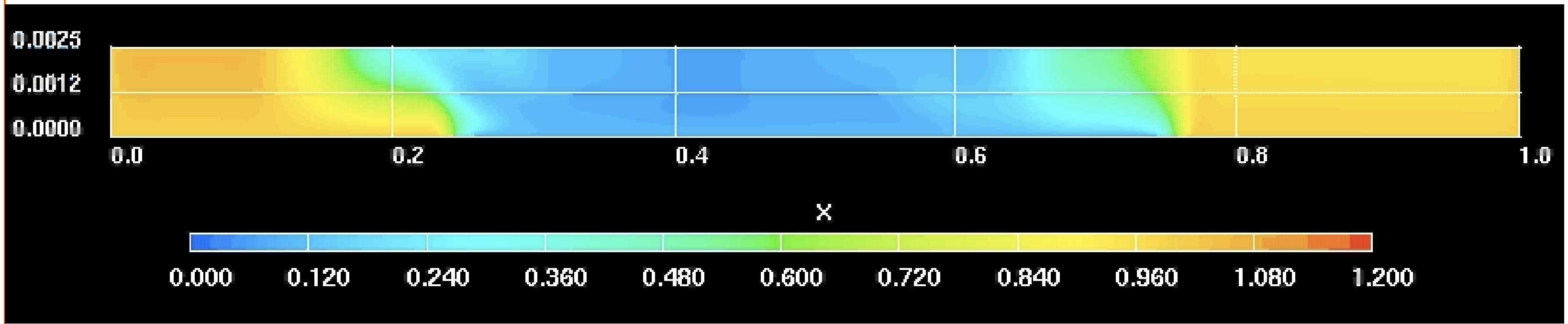
p_0=0.175x10^5 Pa



$\delta_{\alpha \mathrm{H}}/w \approx 2.1$

$\delta_{\alpha \mathrm{C}}/w \approx 0.4$

p_0=1x10^5 Pa

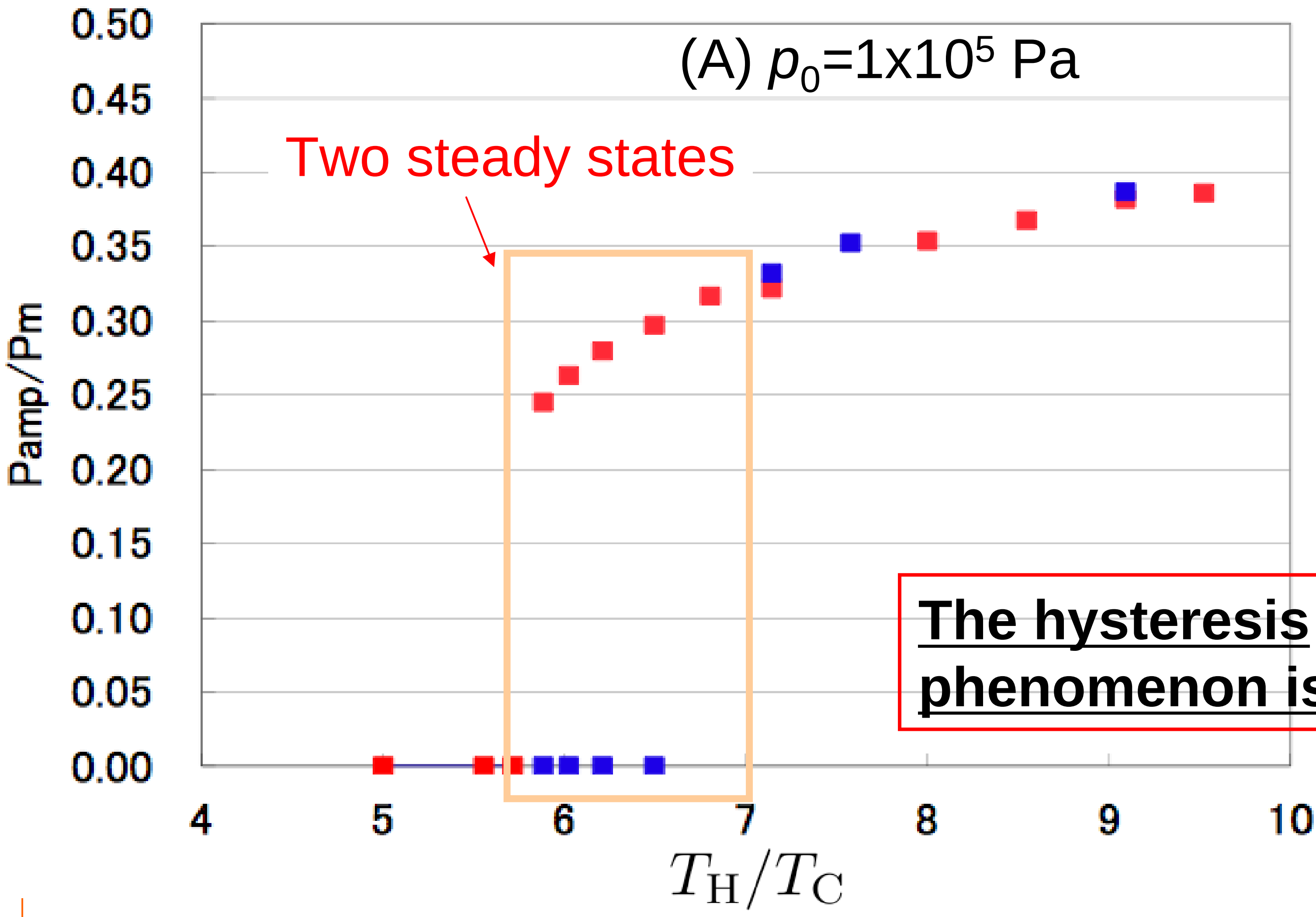


$\delta_{\alpha \mathrm{H}}/w \approx 1.0$

$\delta_{\alpha \mathrm{C}}/w \approx 0.2$

3.3 Pressure amplitude at p_0=1x10^5 Pa

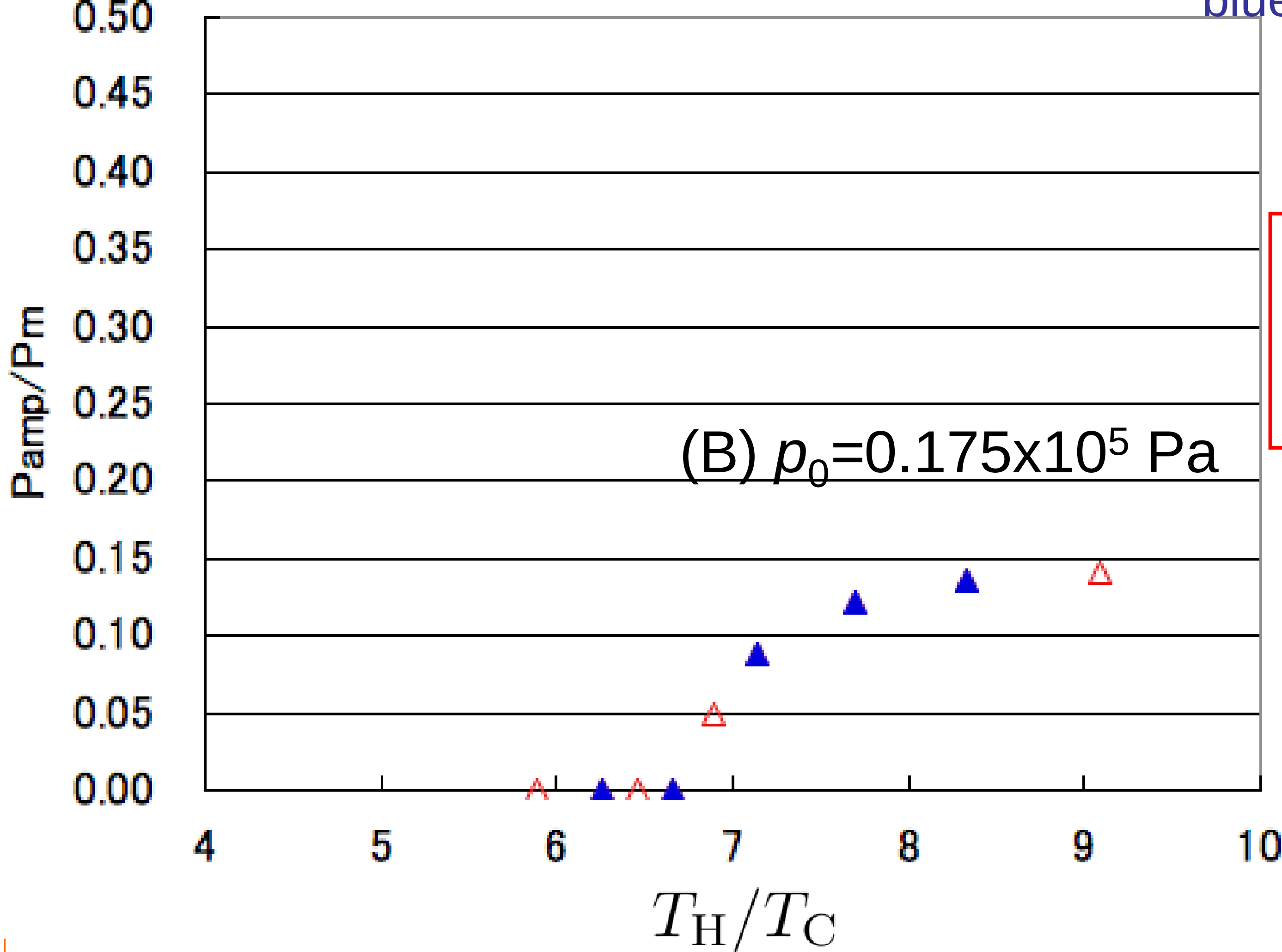
■: P_{amp} (A) p_0=1x10^5 Pa
red : oscillation state initially
blue : quiescent state initially



The hysteresis phenomenon is observed.

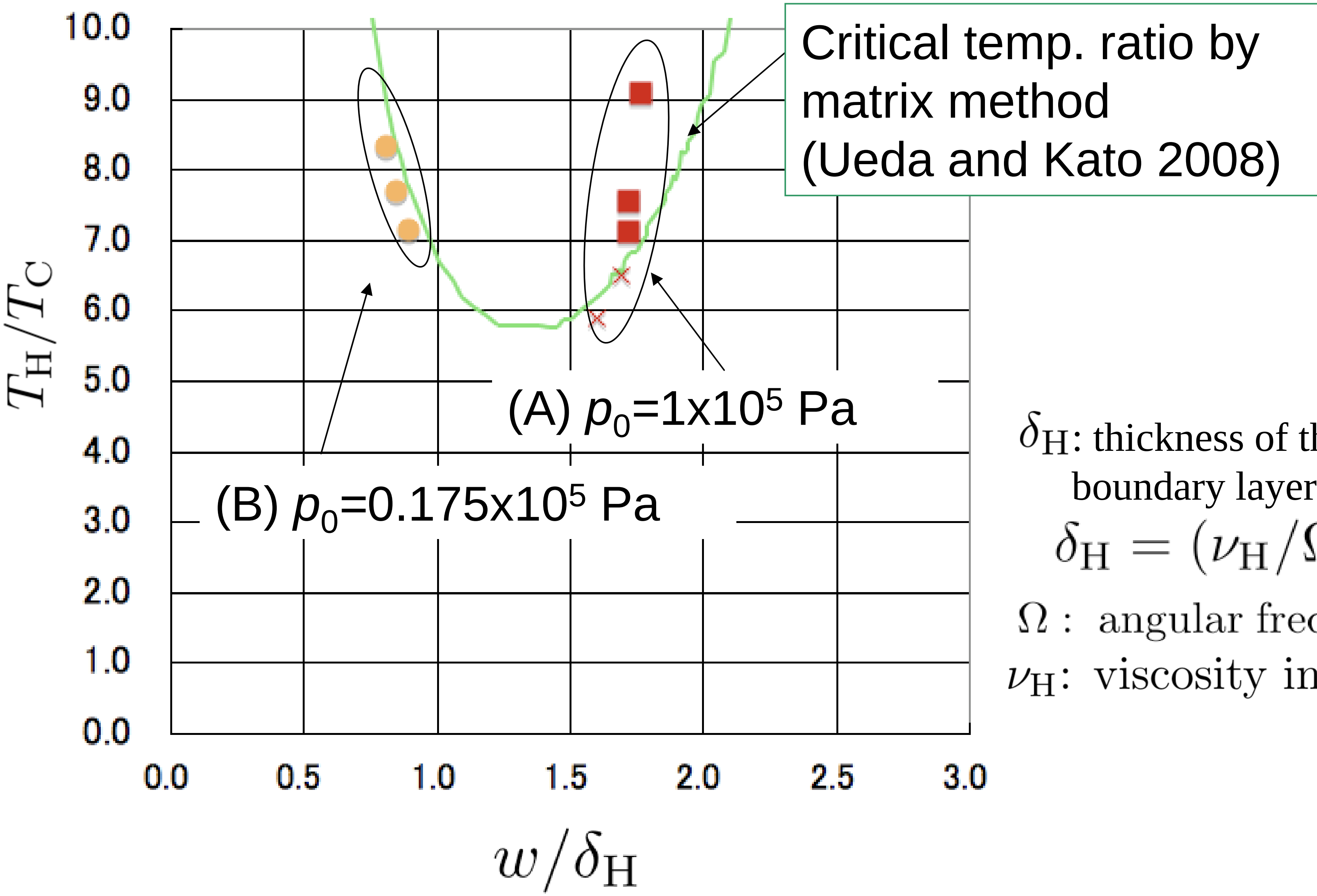
3.3 Pressure amplitude at p_0=0.175x10^5 Pa

▲: P_{amp} (B) p_0=0.175x10^5 Pa
red : oscillation state initially
blue : quiescent state initially



The hysteresis phenomenon is ***not*** observed.

3.4 Critical temperature ratio



δ_H: thickness of the viscous boundary layer

$$\delta_H = (\nu_H/\Omega)^{0.5}$$

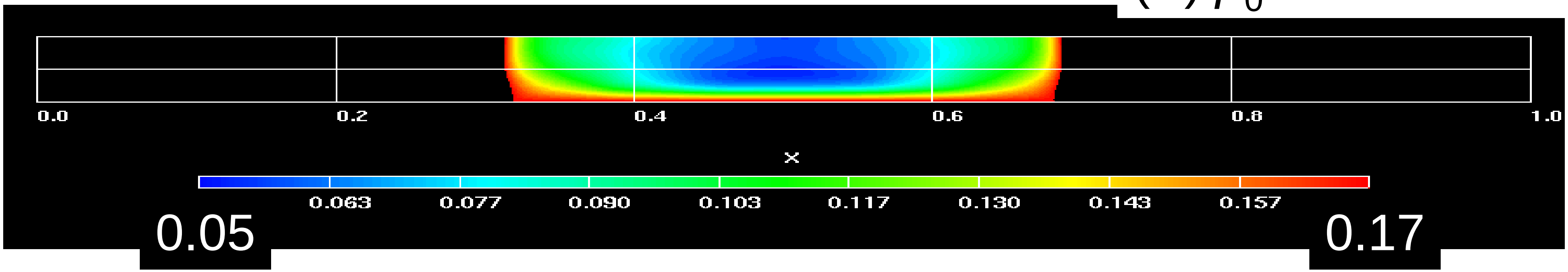
Ω : angular frequency

ν_H: viscosity in hot part

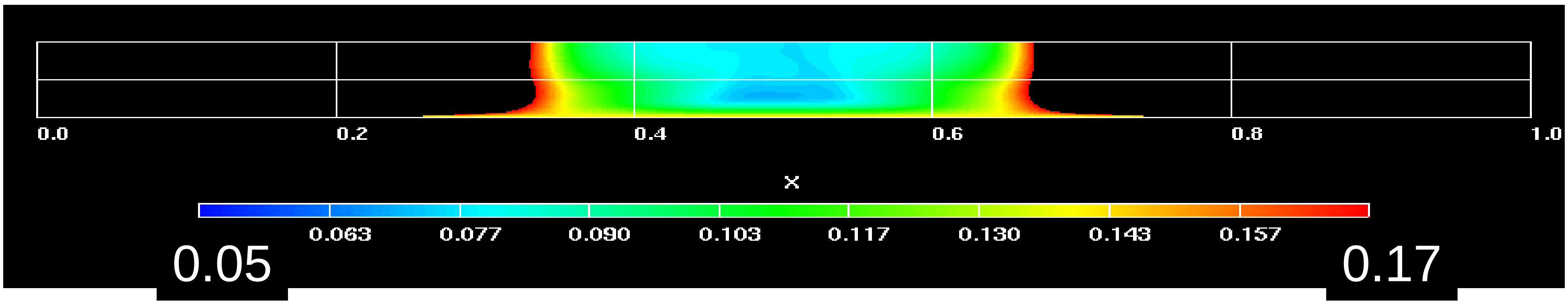
3.5 Time averaged temperature distribution

T_H/T_C=5.88 (T_C=0.17)

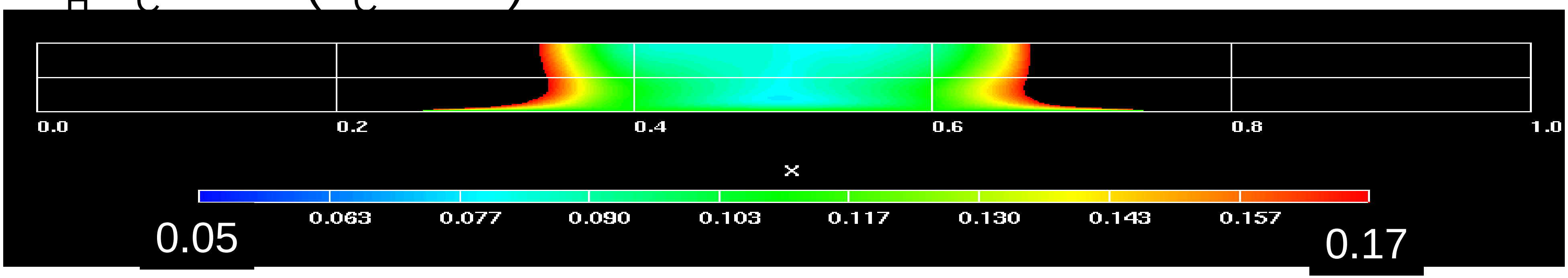
(A) p_0=1x10^5 Pa



T_H/T_C=7.14 (T_C=0.14)

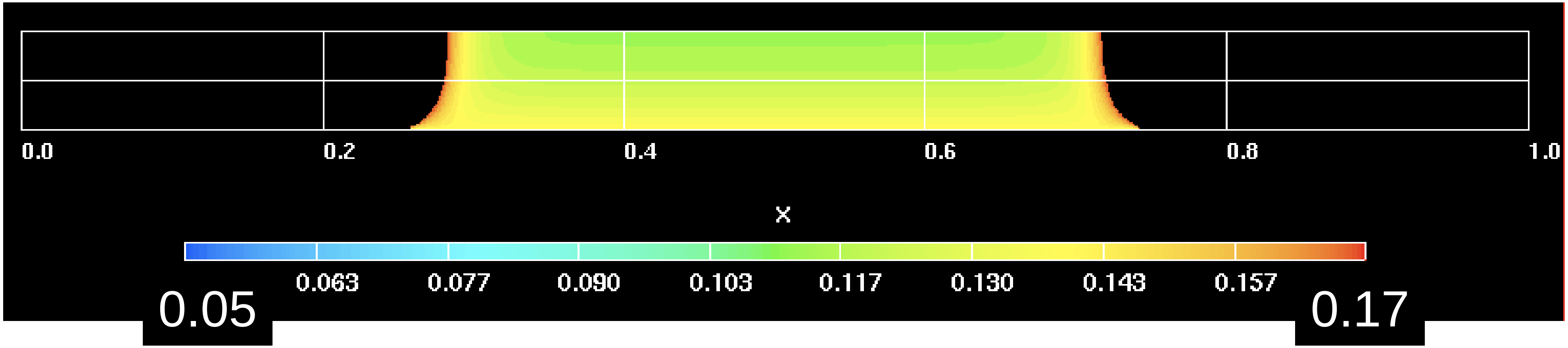


T_H/T_C=9.09 (T_C=0.11)

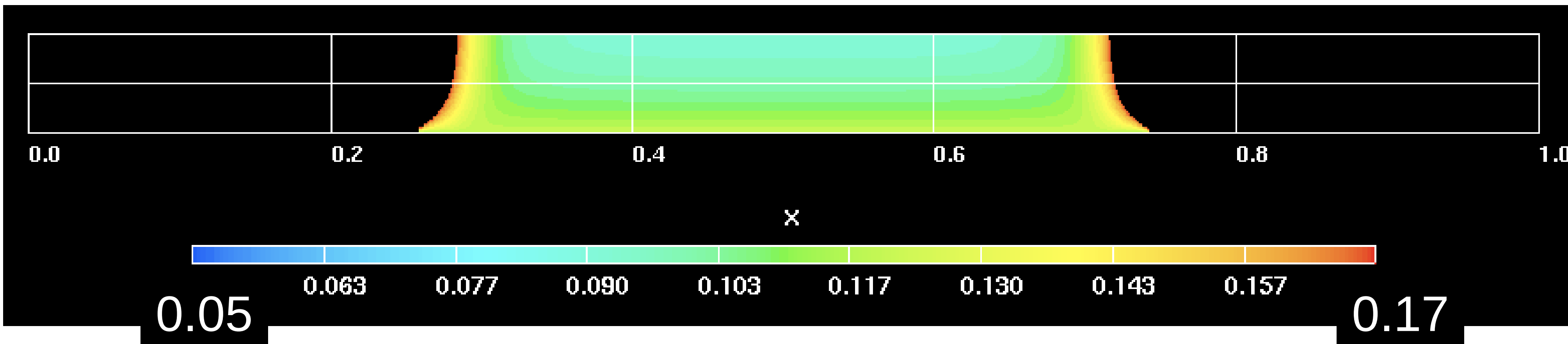


3.5 Time averaged temperature distribution

T_H/T_C=7.14 (T_C=0.14) (B) p_0=0.175x10^5 Pa



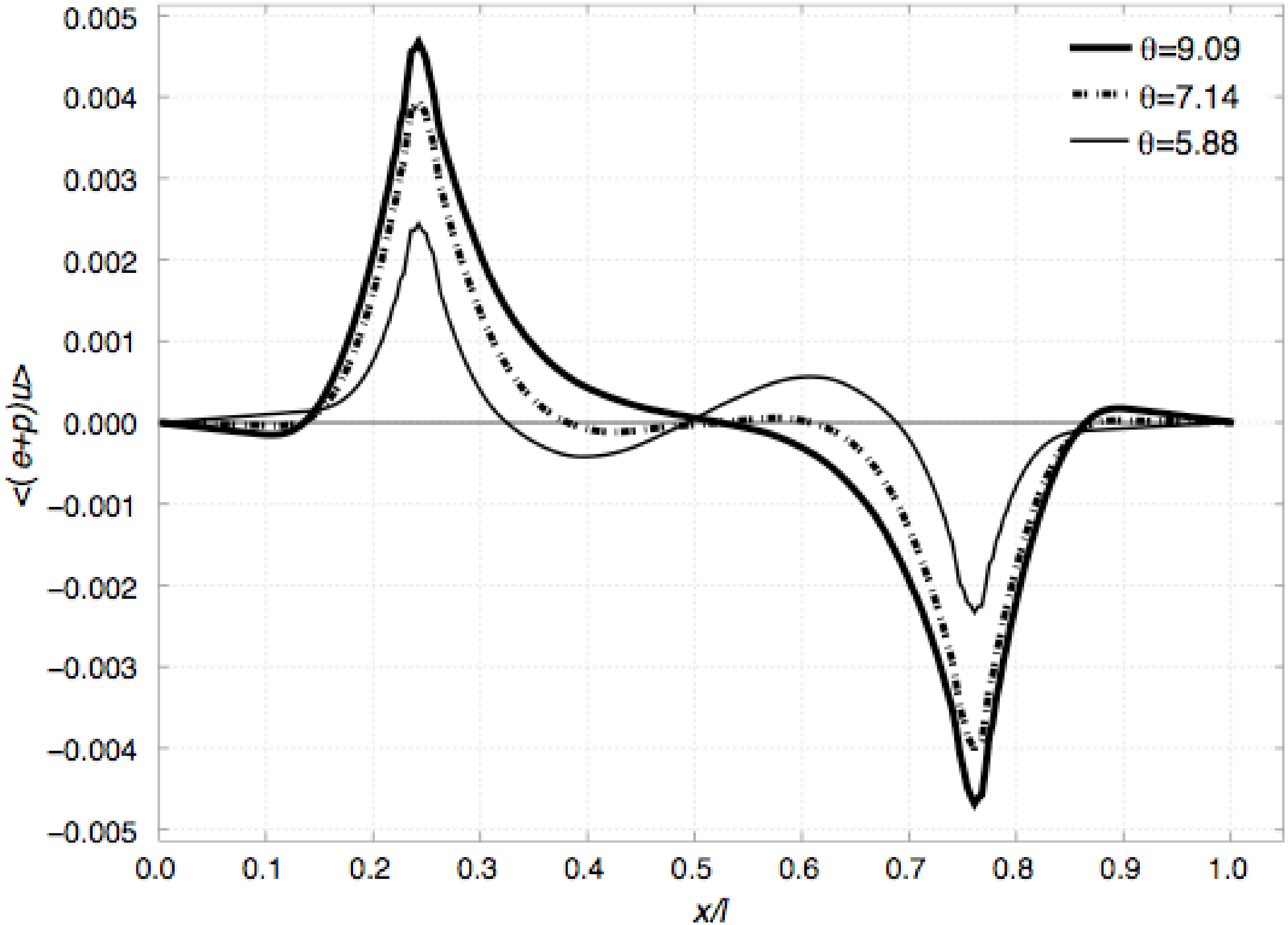
T_H/T_C=8.33 (T_C=0.12)



3.6 Axial energy flux

(A) p_0=1x10^5 Pa

Axial energy flux $= (e+p)u$



4 Summary

- We analyzed the thermo-acoustic oscillation (Taconis oscillation) in the 2D closed tubes by the numerical simulation.

- We observed hysteresis phenomena in the case of the initial pressure $p_0 = 1\text{x}10^5$ Pa. On the other hand, we did not observed the hysteresis phenomenon in the case of $p_0 = 0.175\text{x}10^5$ Pa.

- The heat pumping is observed in the cold region of the tube when the hysteresis phenomenon is observed. On the other hand, the effect of the heat pumping is hardly observed when the hysteresis phenomenon is not observed.