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Vortex Sound: CFD and Experimental Observation

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This is a review paper on vortex sound to commemorate *The 7th International Nobeyama Workshop on CFD* and acknowledge the outstanding contribution to CFD by late Professor Kunio Kuwahara. The lecture starts with a brief introduction on the first scientific work of Kunio Kuwahara.

Perhaps, his first scientific paper in fluid mechanics was the paper with his teacher Isao Imai, *Steady, viscous flow within a circular boundary* [1], published just forty years ago in 1969 in *Phys. of Fluids*. Imai took the part of theoretical formulation of the viscous flow within a circular boundary of radius R at which the velocity $V(\theta)$ is prescribed as a function of angle θ, while Kuwahara's contribution might be both to derive a solution in the form of series expansion with respect to the Reynolds number $Re = UR/\nu$ (correct to Re^8, where U the maximum velocity at the boundary and ν the kinematic viscosity of fluid) and to carry out direct numerical simulation on the HITAC 2050E, the largest computer at that time. The object of this study was twofold: (1) to test the convergence of Stokes' type successive approximation and (2) to have an idea about the limiting form of viscous flow for infinitely large Reynolds number. This case was chosen because it is one of the simplest cases to deal with, and it allows an easy analytical and numerical calculations. Computations were made for $Re = 0, 16, 32, 64, 128, 256$ (Fig.1). They tried even the case $Re = 512, 1024$ with largest number of mesh points 40×40 with 60 min computation (largest time available to normal users). After this brief introduction, we consider main subject of vortex sound.

Any unsteady vortex motion excites acoustic waves. A particular system of two vortex rings is selected in order to clarify its fundamental process and physical mechanism. Experimental detection of the sound wave generated by head-on collision of two vortex rings (Fig.2) was made by Kambe & Minota [2], while a direct numerical simulation (DNS) of the same process (Fig.4) was given by Inoue, Hattori & Sasaki [7] after 17 years from the first experimental detection. Between the two works, there were another studies.

Oblique collision of two vortex rings was investigated theoretically and experimentally by Kambe, Minota & Takaoka [4] in 1993 (Fig.6), and computational study was carried out by Adachi, Ishii & Kambe [5, 6]. In the latter numerical study, the vortex motion was computed by DNS (Fig.7) and the generated sound waves were estimated by using theoretical formulae. Complete DNS of oblique collision of two vortex rings was carried out by Nakashima [8] very recently. In either case of DNS [6, 7, 8], the Reynolds numbers Re were much smaller by 1/7 or 1/40 than the experimental values of about 10^4, where $Re = UR/\nu$ with U being the translation velocity of a single vortex ring and R its ring radius.

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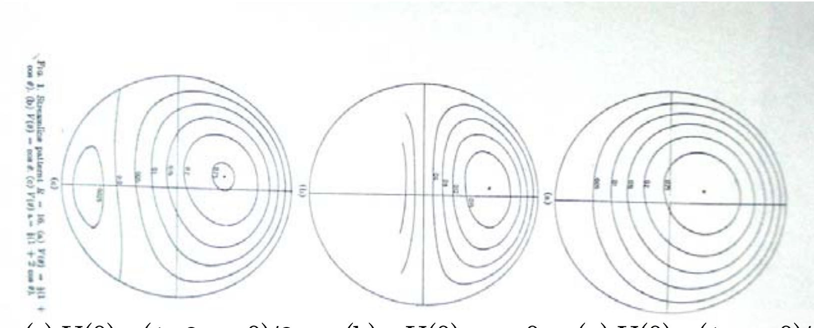
Figures



(c) V(θ)= (1+2 cosθ)/2, (b) V(θ)= cosθ, (a) V(θ)= (1+cosθ)/2.

Fig.1 Streamlines, obtained numerically from 2D NS-equation with Re=16. [1]
(Left half of (b) is the mirror image of the right.)

Head-on Collision of Two Vortex Rings

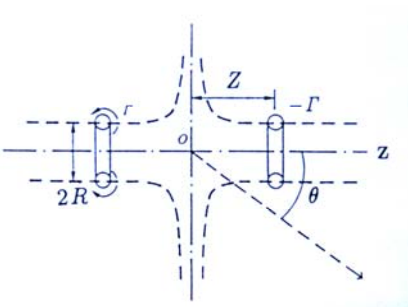


Fig.2 Head-on collision [2].

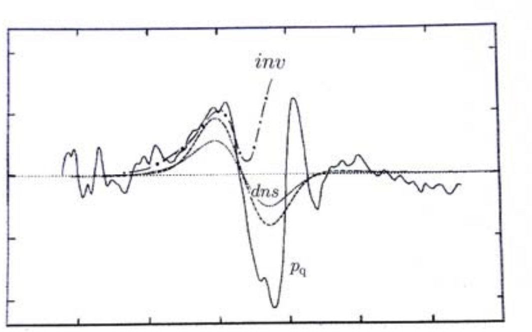


Fig.3 Comparison between four p_q curves [9].

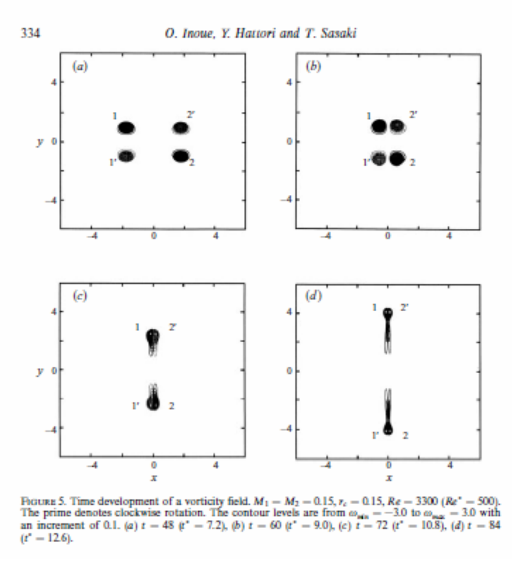


Fig.4 Collision sequence from DNS [7].

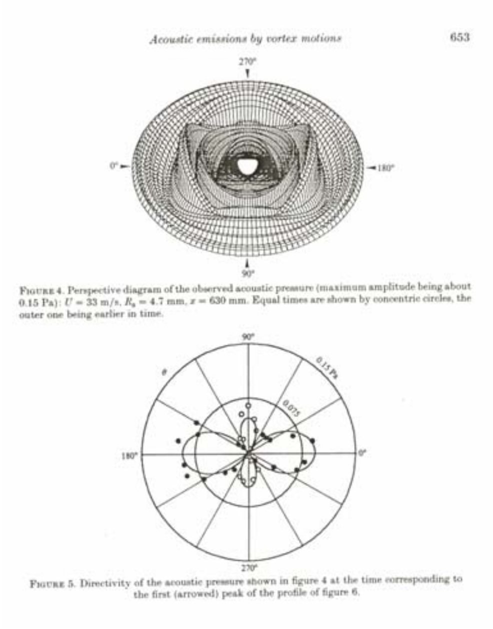


Fig.5 Emission of quadrupolar waves [3].

One can derive the wave equation of *aerodynamic sound* from fundamental equations of fluid mechanics. The wave equation can be reduced to a compact form, called the *equation of vortex sound*. This equation of acoustic pressure predicts sound generation by unsteady vortex motions. One can derive a formula of wave pressure excited by time evolution of vorticity field (which is localized in space) [9].

Axisymmetric head-on collision

The wave emitted by an axisymmetric head-on collision of two vortex rings (Fig.5) is characterized by quadrupolar directionality : $p_{\rm q}(t)(1 - 3\,\cos^2\theta)$, where θ is the angle from the symmetry axis and $p_{\rm q}(t)$ is the wave amplitude at a time at $\theta = \pi/2$. However, in a viscous fluid like the air, there is another monopolar component $p_{\rm m}(t)$. Thus, the observed pressure of the wave is represented as

$$p(r_*, t) = p_{\rm m}(t) + p_{\rm q}(t)\,(1 - 3\,\cos^2\theta), \tag{1}$$

Figure 3 shows comparison between four curves of $p_{\rm q}(t)$: observed $p_{\rm q}(t)$ (*solid curve*), theoretical inviscid profile *inv* (*chain-dotted*), and DNS curves *dns* (two *broken curves* for two different *Re*-values). Existence of the monopole component $p_{\rm m}(t)$ is also confirmed by DNS [7].

Oblique collision

At the time of oblique collision of two vortex rings, there is a process of vortex reconnection, which emits a characteristic pulse of particular directivity (Fig.8). Observed trajectories of the vortex cores are shown by dots in Fig.6. Figure 7 shows a time sequence of oblique collision obtained by DNS [6] with side view (left) and top view (right). The morphologies depicted are defined by the isovorticity surfaces of 40% of the maximum vorticity value.

Symmetry consideration leads to the following expression for the far-field acoustic pressure, to be observed at a fixed radial distance r from the origin:

$$\begin{aligned} p(\theta,\phi,t)\Big|_r &= A_0(t) + A_1(t)\,P_2^0(\cos\theta) + A_2(t)\,P_2^2(\cos\theta)\cos 2\phi \\ &\quad + B_1(t)\,P_3^0(\cos\theta) + B_2(t)\,P_3^2(\cos\theta)\cos 2\phi\,, \end{aligned} \tag{2}$$

where P_n^m are the Legendre polynomials, θ and ϕ are the angle variables of the spherical coordinates (shown up to $n = 3$), and higher order terms are omitted since observed mode amplitudes were found to be insignificant. The first two terms correspond to (1). There are five amplitude functions,

$$A_0(t),\ A_1(t),\ A_2(t),\ B_1(t),\ B_2(t).$$

These are determined from the observed signals. The five mode amplitudes were calculated from the DNS data and compared with those of experimental signals [4, 8] (Fig.9). Both results showed good qualitative agreement.

Comparison of the generated wave profiles shows excellent agreement between those of DNS and observed ones, although Reynolds numbers and Mach numbers were different between them. These are consistent with the theoretical predictions too.

Oblique Collision of Two Vortex Rings

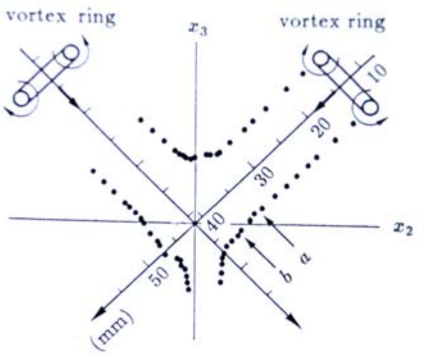


Fig.6 Trajectories of vortex cores [4].

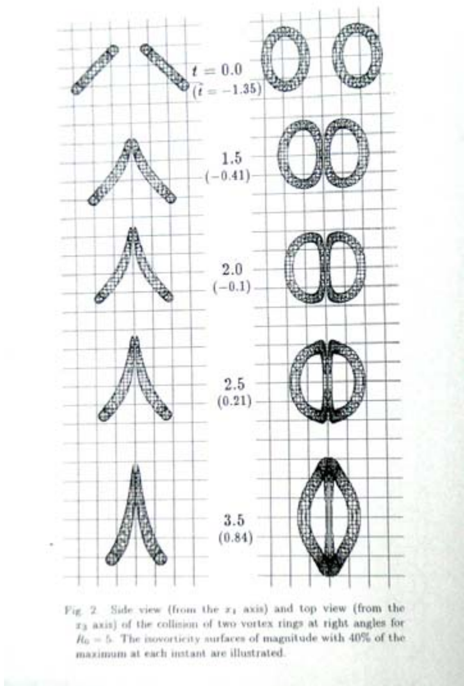


Fig.7 Collision sequence from DNS [5, 6].

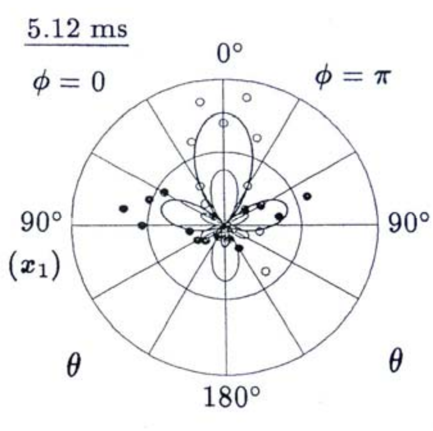


Fig.8 Asymmetric emission [4].

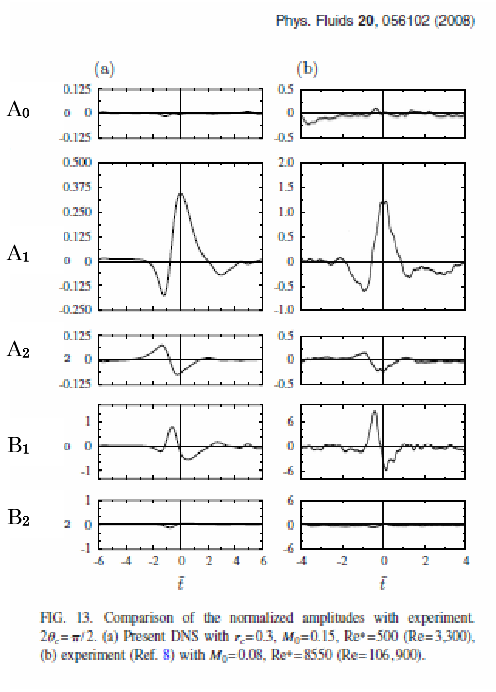


Fig.9 Comparison of wave profiles of five components [8].

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