How to Model Turbulent Dissipation?

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Tokyo, September 24th 2009

Fully-developed turbulence

Reynolds number Re is the ratio between
the norm of the convection and vortex stretching terms
and the norm of the dissipation term.

$$\partial_t \vec{\omega} \; + \; \boxed{\vec{V} \cdot \nabla \vec{\omega}} \; - \; \boxed{\vec{\omega} \cdot \nabla \vec{V}} \; = \; \boxed{\nu \nabla^2 \vec{\omega}}$$

Fully-developed turbulence regime when Reynolds number is very large, *i.e.*, **convection strongly dominates viscous dissipation**.

Fully-developed turbulent flow properties :

- **sensitivity to initial conditions**

$$\Longrightarrow$$

deterministic unpredictability,

- **mixing**

$$\Longrightarrow$$

statistical predictability,

- **dissipation** becomes **independent of Re,**
i.e., of viscosity.

How to decompose turbulent fluctuations?

'In 1938 Tollmien and Prandtl suggested that turbulent fluctuations might consist of two components, a diffusive and a non-diffusive. Their ideas that fluctuations include both random and non random elements are correct, but as yet there is no known procedure for separating them.'

Hugh Dryden, Adv. Appl. Mech., 1, 1948

turbulent fluctuations
= **non random** + **random**
= **coherent structures** + **incoherent noise**

turbulent dynamics
= chaotic **non diffusive** + stochastic **diffusive**
= **inviscid nonlinear** dynamics + **turbulent dissipation**

$\Longrightarrow$ Coherent Vorticity Simulation (CVS)

*Farge et al., Fluid Dyn. Res., **10**, 229, 1992*

*Farge, Schneider, Kevlahan, Phys. Fluids, **11** (8), 1999*

*Farge, Pellegrino, Schneider Phys. Rev, Lett. **87** (5), 2001*

How to extract coherent fluctuations?

Since there is not yet a universal definition of coherent fluctuations (*i.e.*, coherent structures), we adopt an **apophetic method** :
instead of defining what they are, we define what they are not.

We propose the minimal statement:
'Coherent fluctuations are not noise'

$\Longrightarrow$

Extracting coherent fluctuations becomes a **denoising problem**, not requiring any hypotheses on the coherent structures but only on the noise to be eliminated.

Choosing the **simplest hypothesis** as a first guess, we propose to eliminate an additive Gaussian white noise, and use **nonlinear wavelet filtering** for this.

Farge et al., Fluid Dyn. Res., 10, 229, 1992

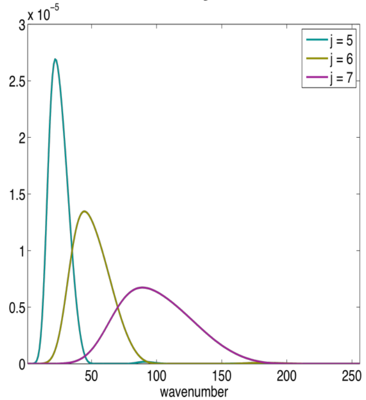
Farge, Schneider, Kevlahan, Phys. Fluids, 11 (8), 1999

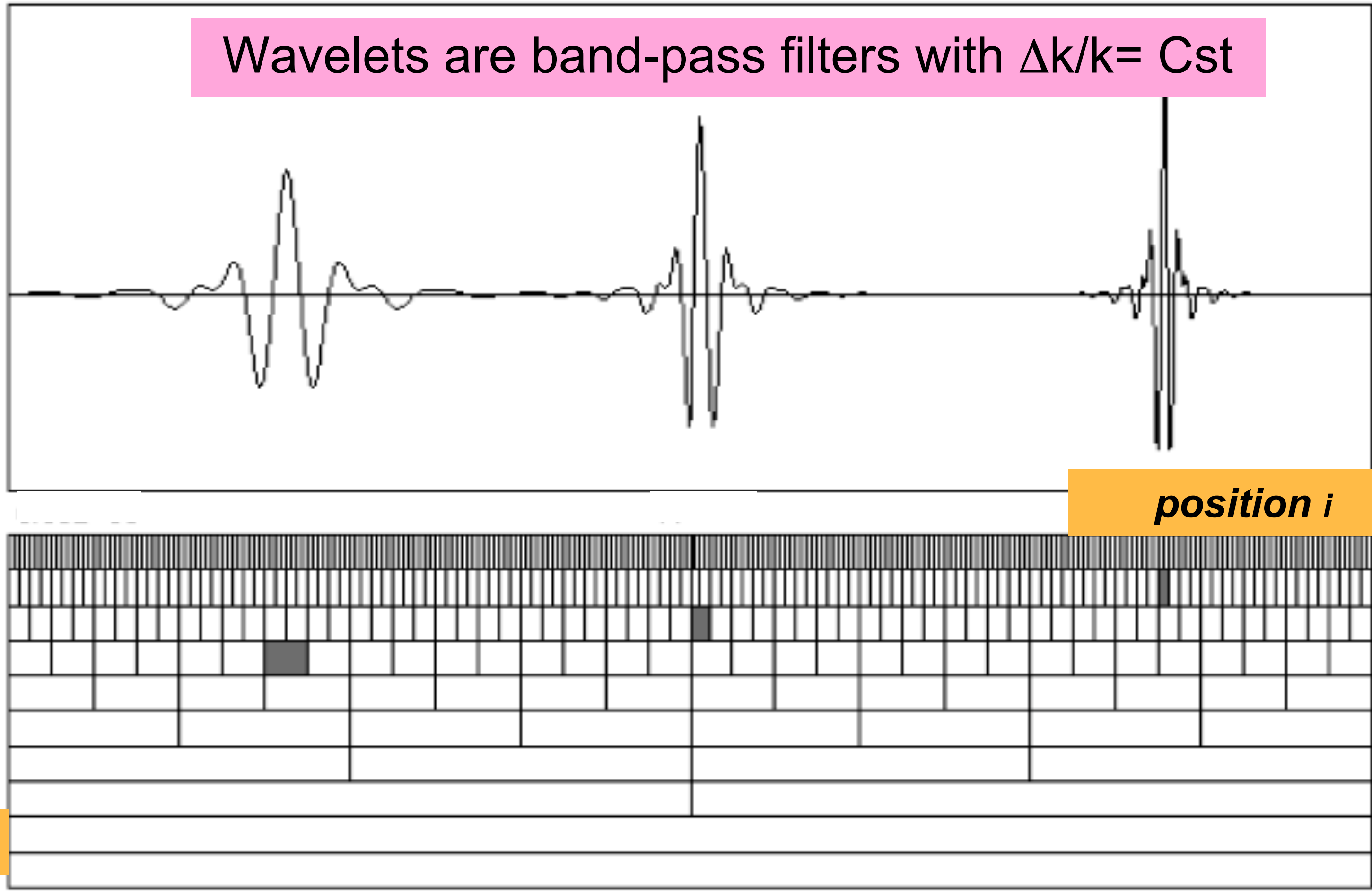
Farge, Schneider et al. Phys. Fluids, 15 (10), 2003

Wavelet representation

Wavelets

$$\psi_{ji}$$





Wavelet coefficients

$$\tilde{f}_{ji} = \langle \psi_{ji} | f \rangle$$

*A. Grossmann and J. Morlet, Decomposition of Hardy functions into square integrable wavelets of constant shape, SIAM J. Math. Anal., **15**, 1984*

*M. Farge, Wavelet transforms and their applications to turbulence, Ann. Rev. Fluid Mech., **92**, 1992*

Wavelet-based denoising

1. Goal:

   Extraction of coherent vortices
   from a noise which will then be modelled
   to compute the flow evolution.

2. Apophatic principle:

   - **no hypothesis on the vortices**,
   - **only hypothesis on the noise**,
   - simplest hypothesis as our first choice.

3. Hypothesis on the noise:

   $f_B = f + w$

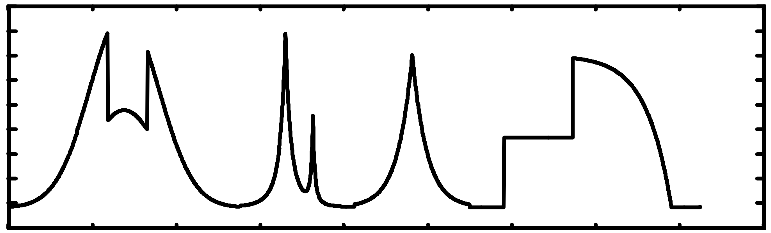
   w : Gaussian white noise,
   σ^2 : variance of the noise,
   N : number of coefficients.

4. Computation of the threshold:

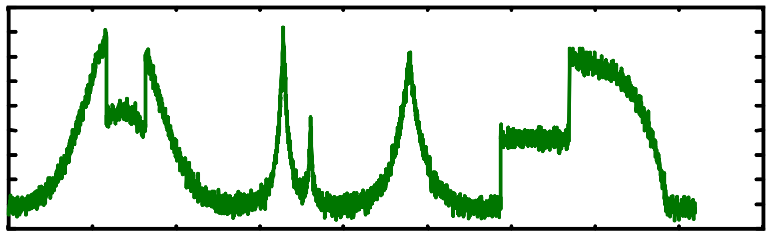
$$\varepsilon_D = \sqrt{2\sigma^2 \ln(N)}$$

5. Denoised signal:

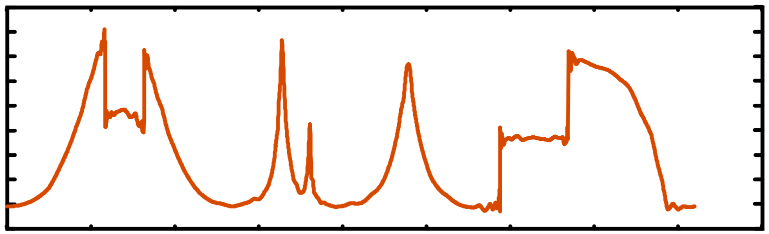
$$f_D = \sum_{\lambda : \left|\tilde{f}_\lambda\right| < \varepsilon} \tilde{f}_\lambda \psi_\lambda$$



f



f_B

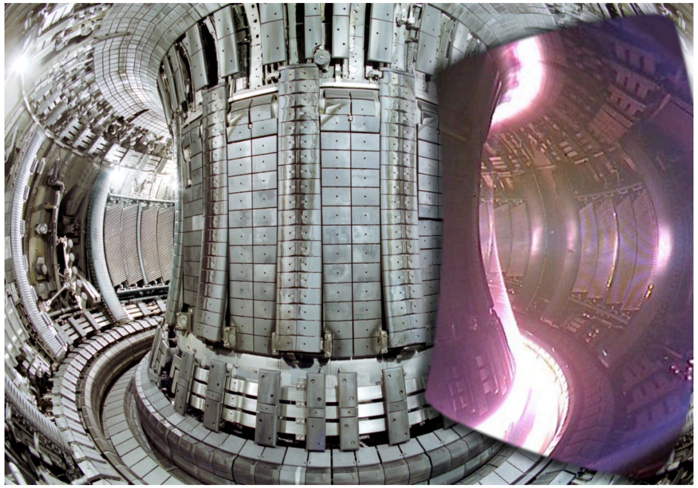


f_D

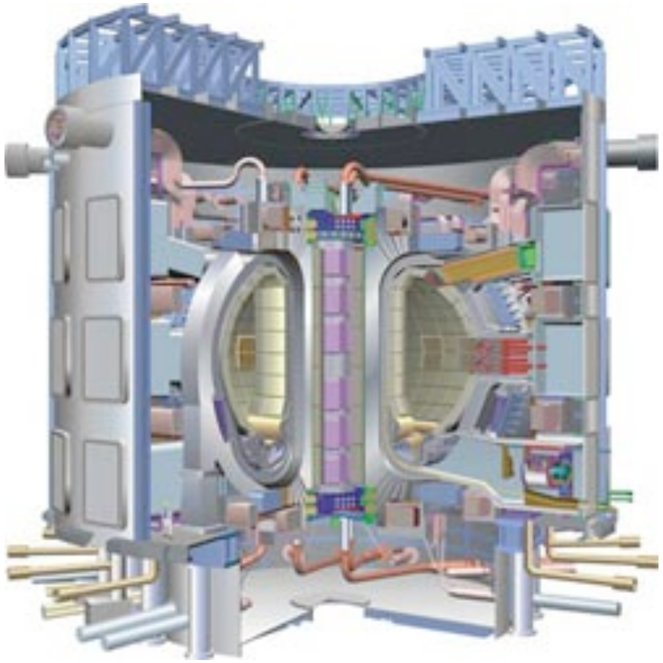
Donoho and Johnstone
*Biometrika, **81**, 1994*

Azzalini, Farge and Schneider
*ACHA, **18** (2), 2005*

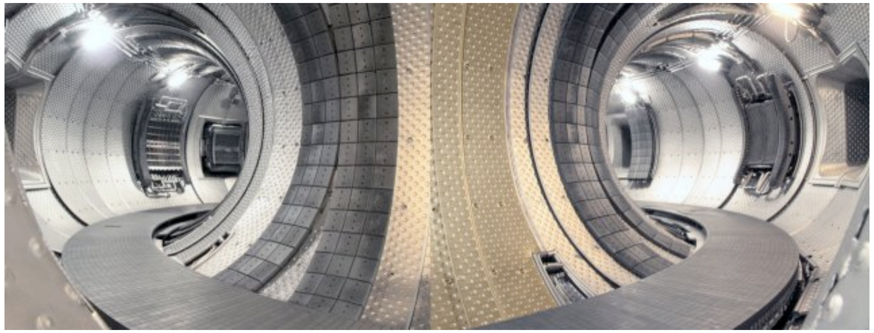
Application to edge plasma in tokamaks



JET, Culham (UK)



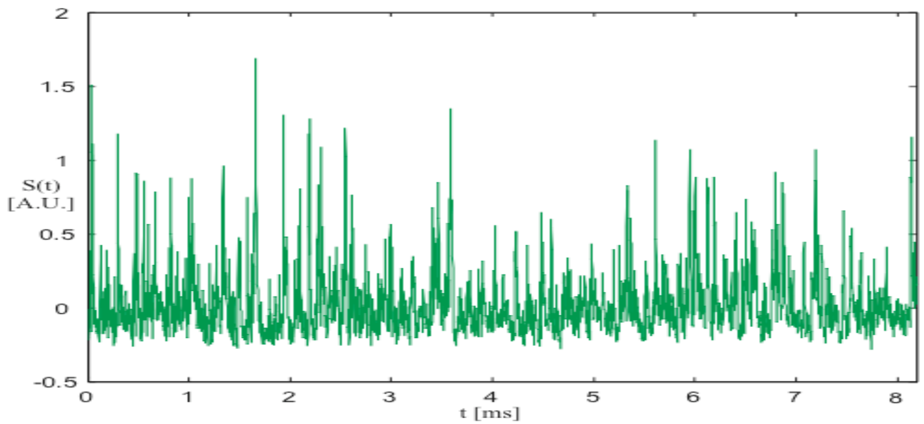
ITER (2015)

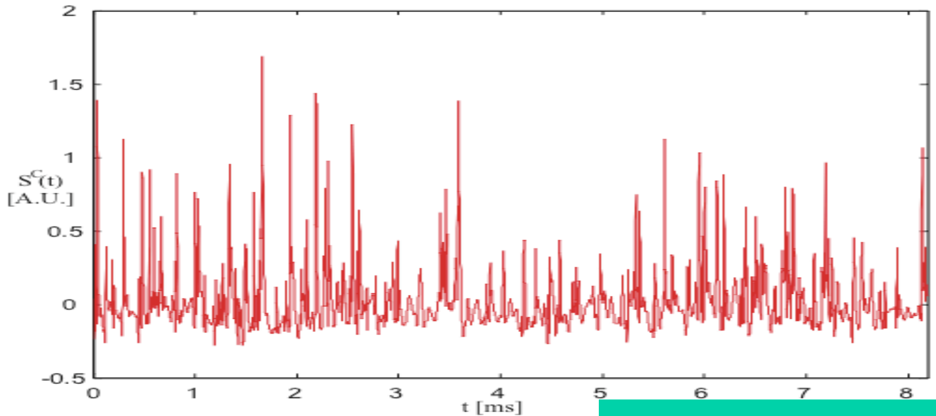


Tore-Supra, Cadarache (France)

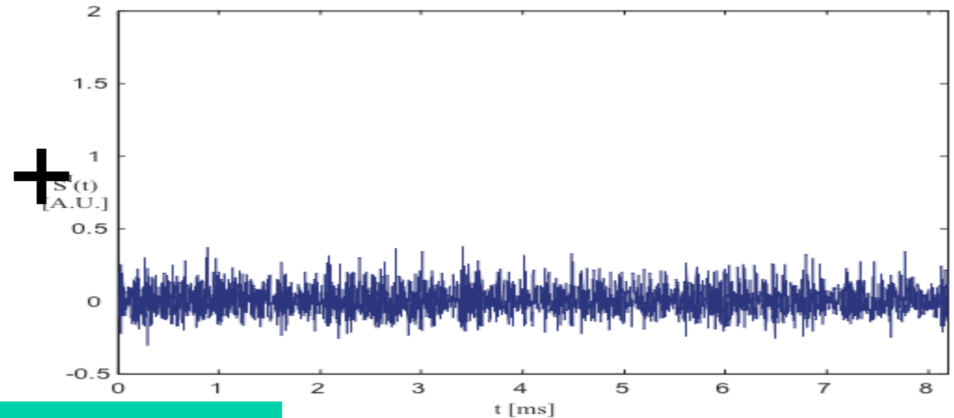
Wavelet-denoising Tore-Supra edge plasma

Saturation current measured by a Langmuir probe
in the scrape-off layer of the tokamak Tore Supra
(Pascal Devynck, Tore-Supra, CEA-Cadarache)





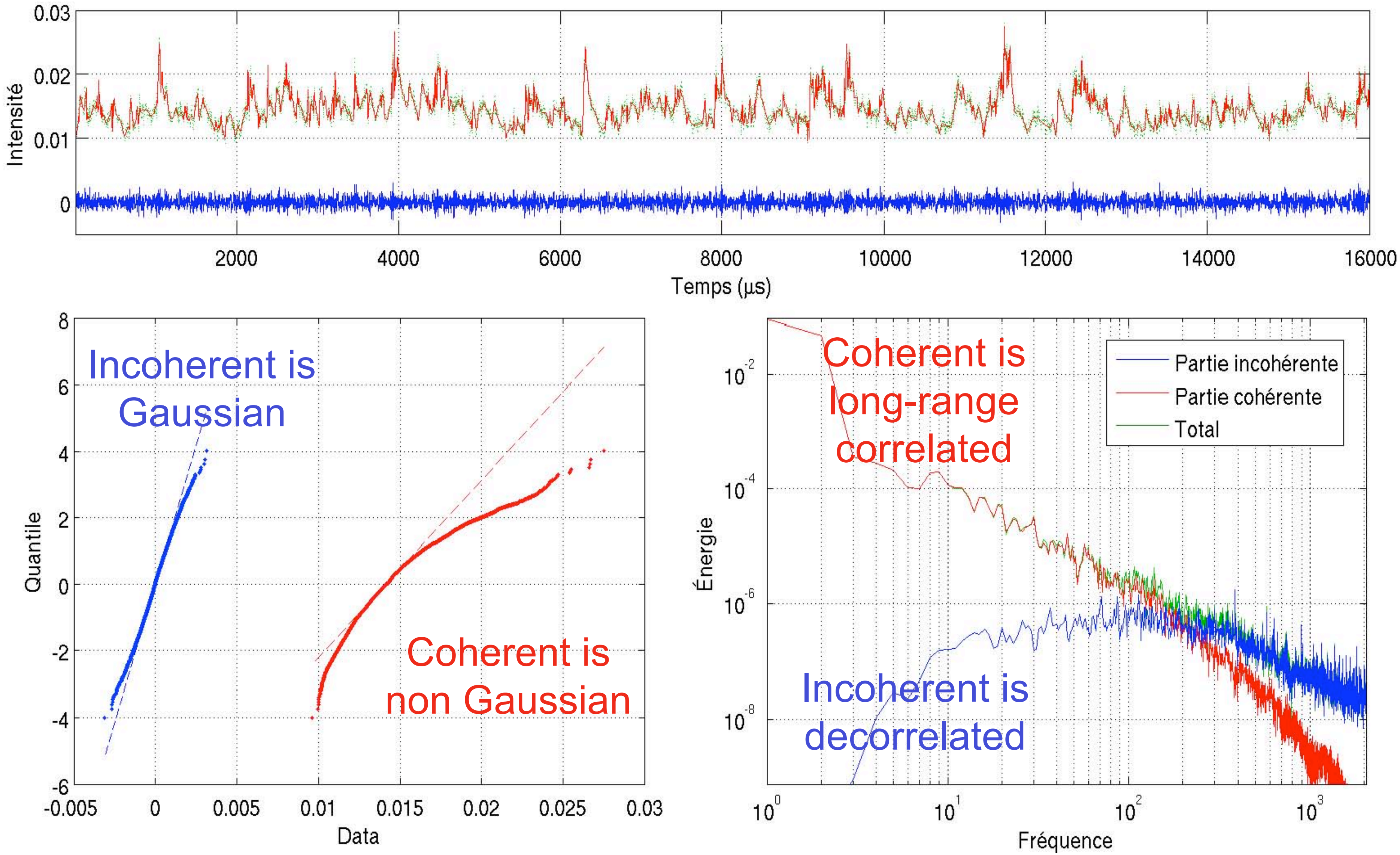
Coherent bursts



Incoherent background

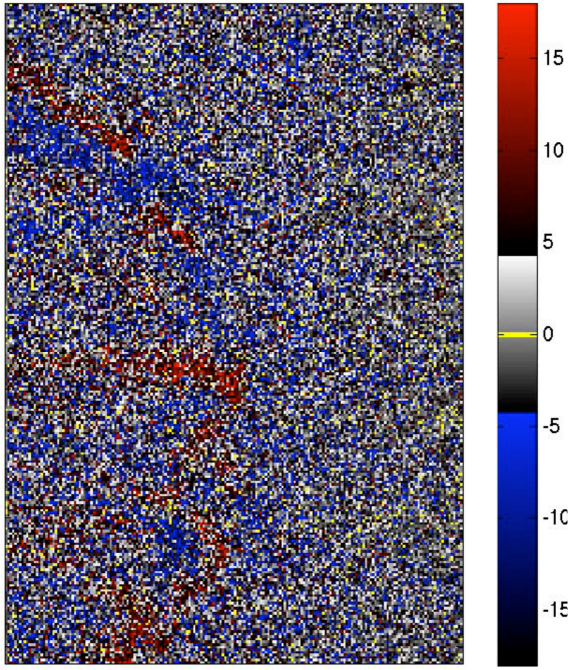
Farge, Schneider & Devynck
Phys. Plasmas,13, 042304, 2006

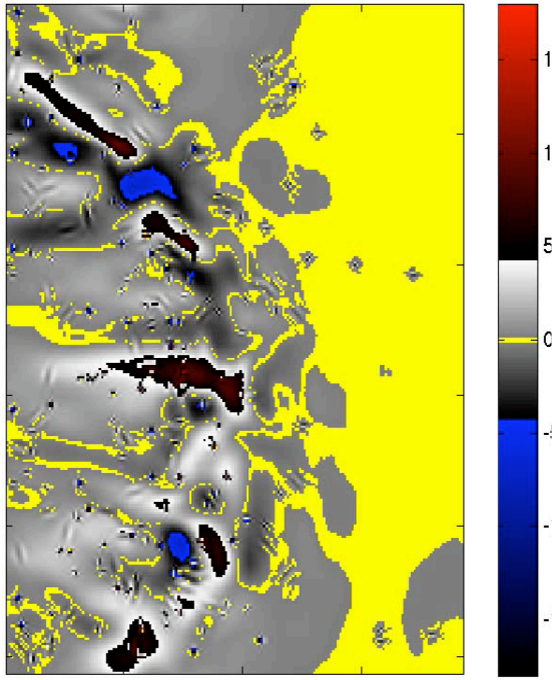
Wavelet-denoising of density fluctuations

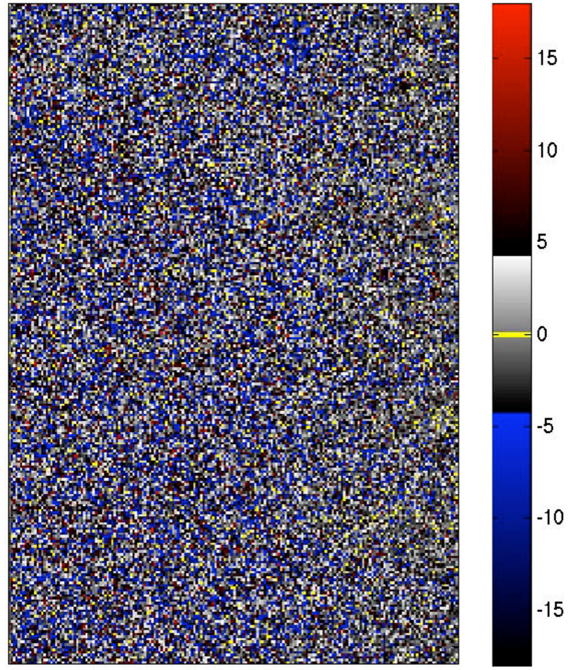


Visible light fast camera in Tore-Supra

(Pascale Garbet, Gérard Bonhomme, Nicolas Fedorczak)







Total

Coherent

Incoherent

Coherent Vorticity Extraction (CVE)

- Vorticity $\vec{\omega} = \nabla \times \vec{v}$ at resolution $N = 2^{3J}$

The threshold could also be estimated independently for each scale and for each direction

- Wavelet transform $\tilde{\vec{\omega}} = \langle \vec{\omega}, \psi_\lambda \rangle$

- Thresholding: $T = (4/3 Z \ln N)^{1/2}$

$$\tilde{\vec{\omega}}_C = \begin{cases} \tilde{\vec{\omega}} & \text{for} \quad |\tilde{\vec{\omega}}| \geq T, \\ 0 & \text{for} \quad |\tilde{\vec{\omega}}| < T \end{cases} \qquad \tilde{\vec{\omega}}_I = \begin{cases} \tilde{\vec{\omega}} & \text{for} \quad |\tilde{\vec{\omega}}| < T, \\ 0 & \text{for} \quad |\tilde{\vec{\omega}}| \geq T \end{cases}$$

Iterative procedure to estimate the noise from the weak wavelet coefficients

- Inverse wavelet transform to reconstruct $\vec{\omega}_C + \vec{\omega}_I = \vec{\omega}$

- Apply Biot-Savart operator to reconstruct $\vec{v}_C + \vec{v}_I = \vec{v}$ with $\vec{v} = \nabla \times \nabla^{-2} \vec{\omega}$

- Remark: $Z = Z_C + Z_I$ (orth. dec.) and $E \approx E_C + E_I$

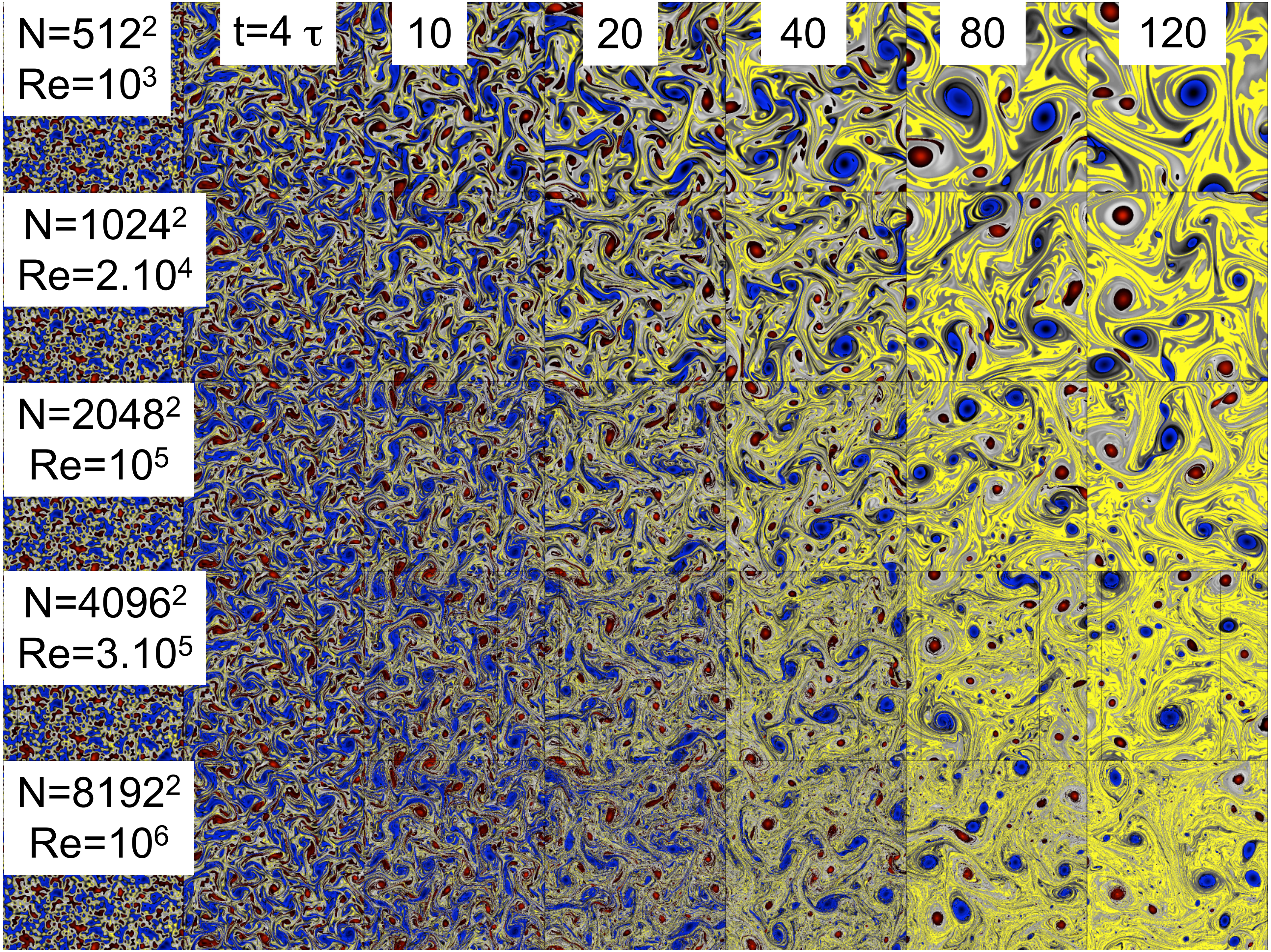
- Linear complexity, $\mathcal{O}(\mathcal{N})$

Azzalini, Farge, Schneider
Appl. Comp. Harmonic. Anal., ***18***(2), 2005

Comparison between
CVS-regularized 2D Euler
and 2D Navier-Stokes
in a periodic domain

Comparison between
CVS-regularized 2D Euler
and 2D Navier-Stokes
in a bounded circular domain

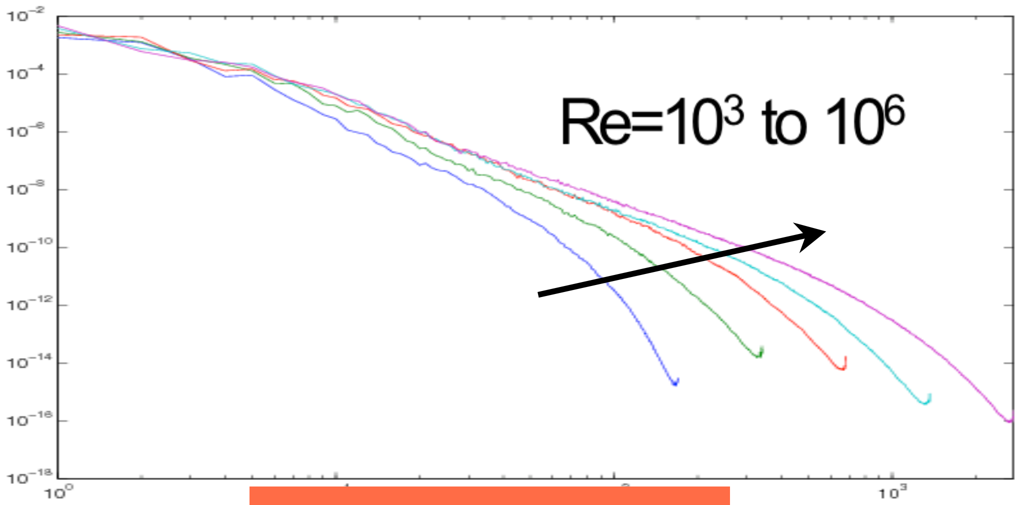
Comparison between
CVS-regularized 3D Euler
and 3D Navier-Stokes
in a periodic domain



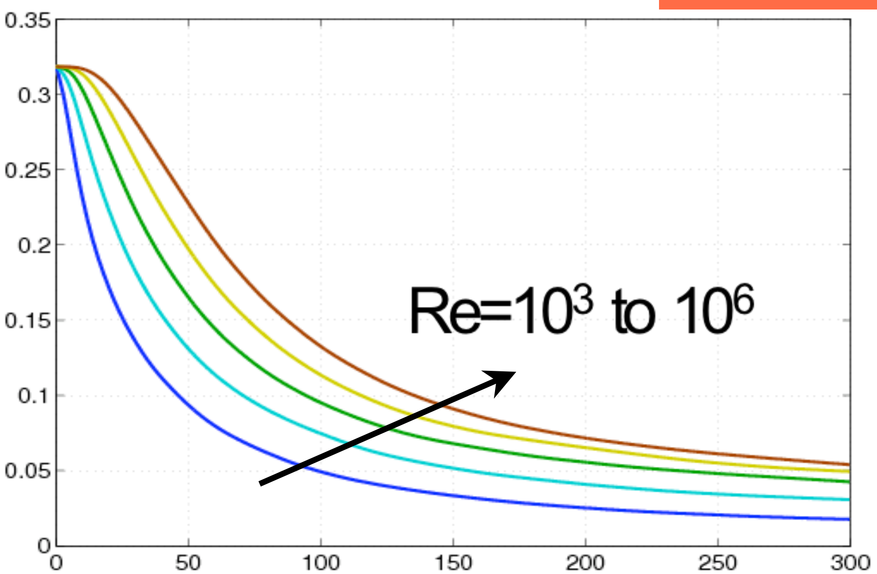
N=512²
Re=10³
t=4 τ
10
20
40
80
120
N=1024²
Re=2.10⁴
N=2048²
Re=10⁵
N=4096²
Re=3.10⁵
N=8192²
Re=10⁶

5 DNSs
from $N=512^2$
to $N=8192^2$

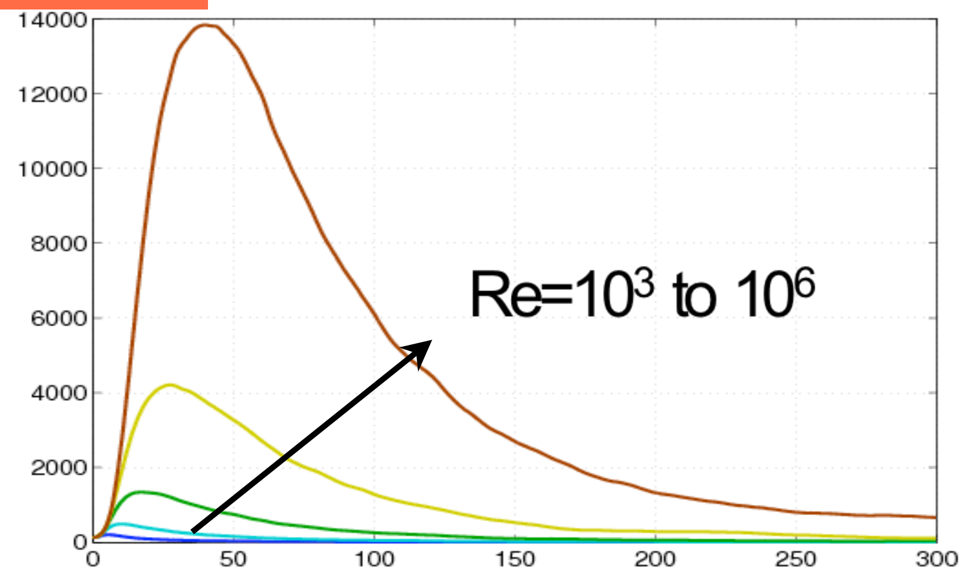
2D viscous Navier-Stokes with periodic boundary conditions



Enstrophy spectrum



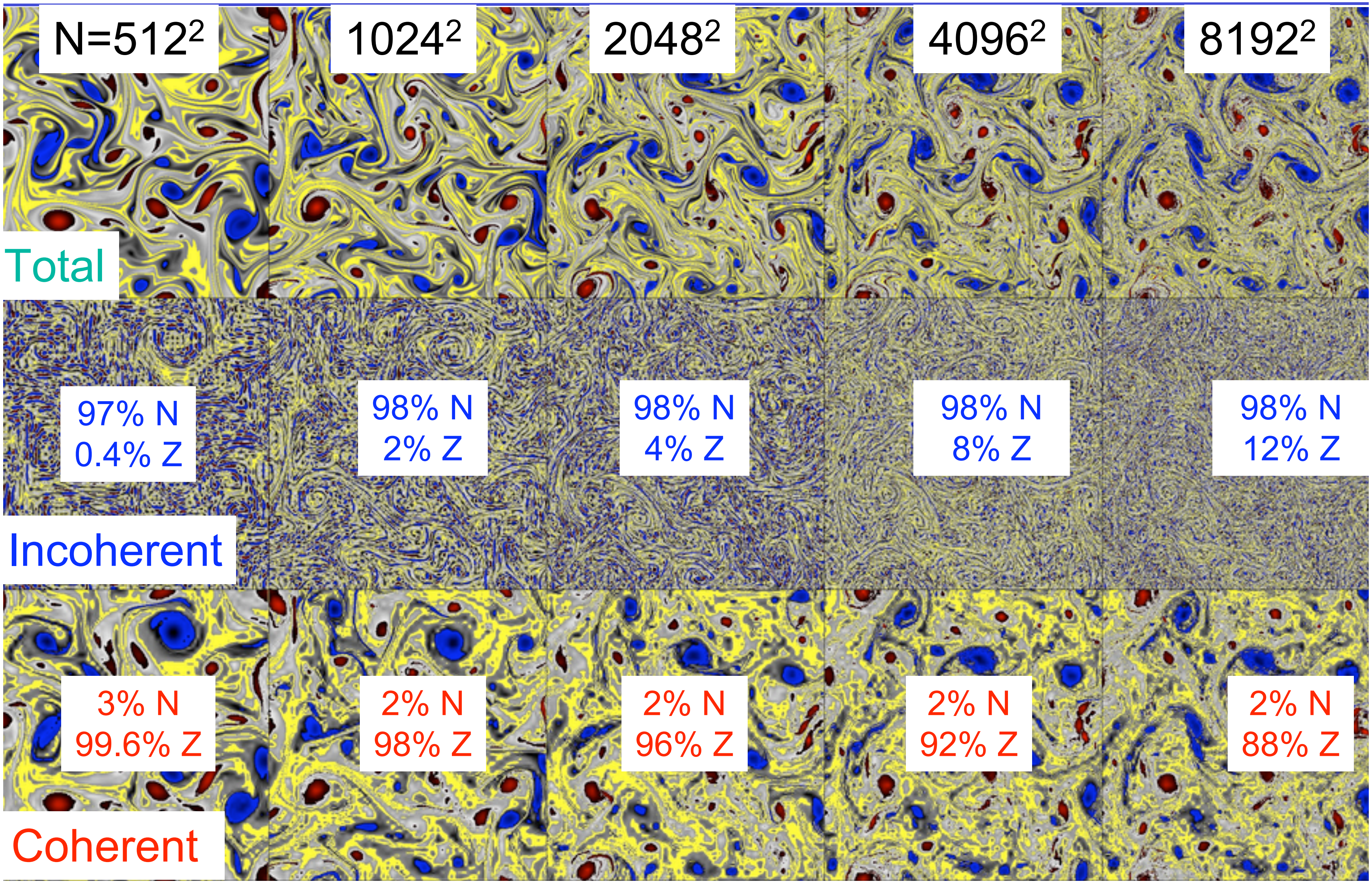
Time evolution of enstrophy



Time evolution of palinstrophy

$t=40\ \tau$

Coherent Vorticity Extraction



Coherent Vorticity Extraction

$N=8192^2$
$t=120\ \tau$

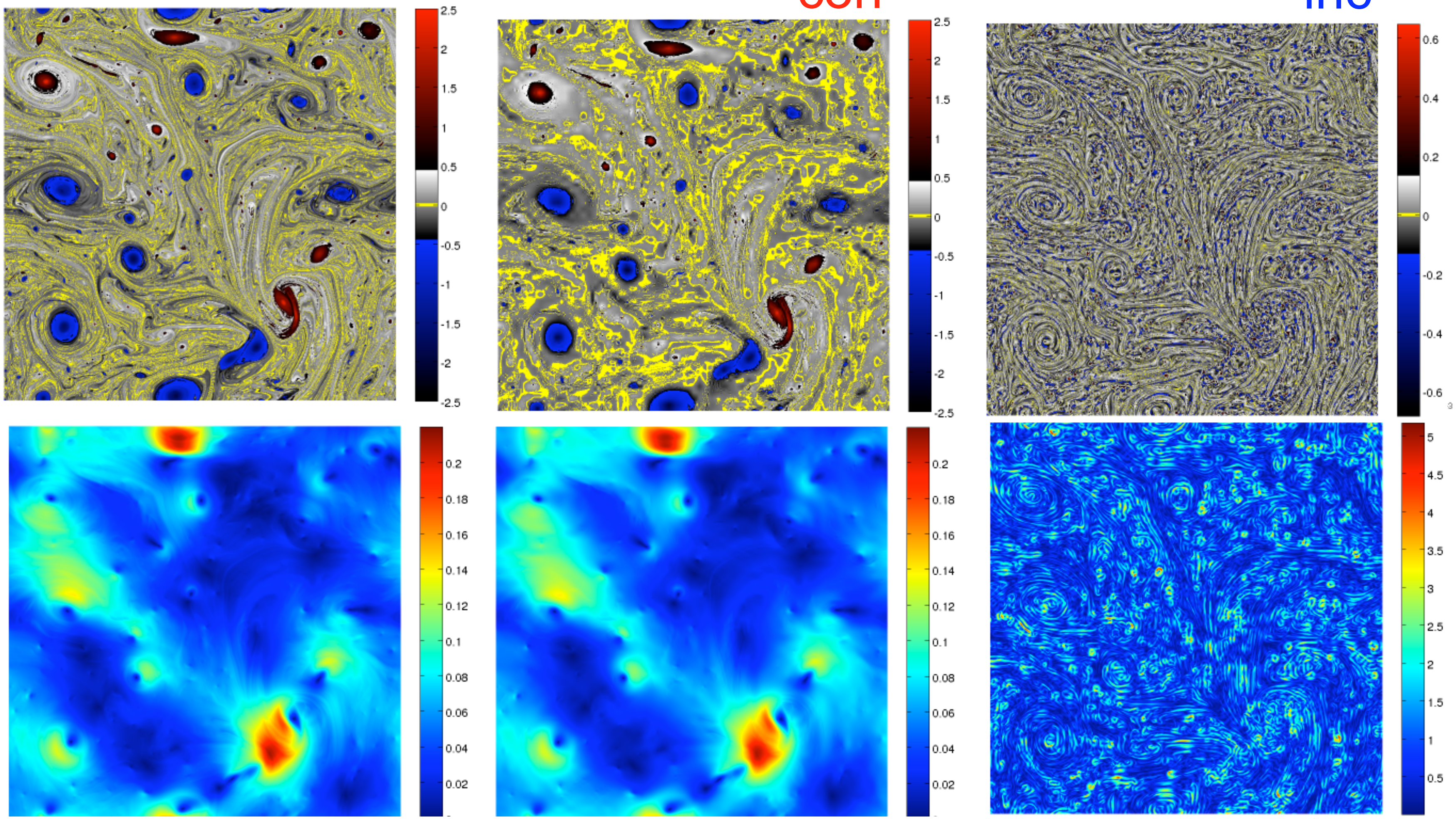
tot

2% N
88% Z
99.9% E

coh

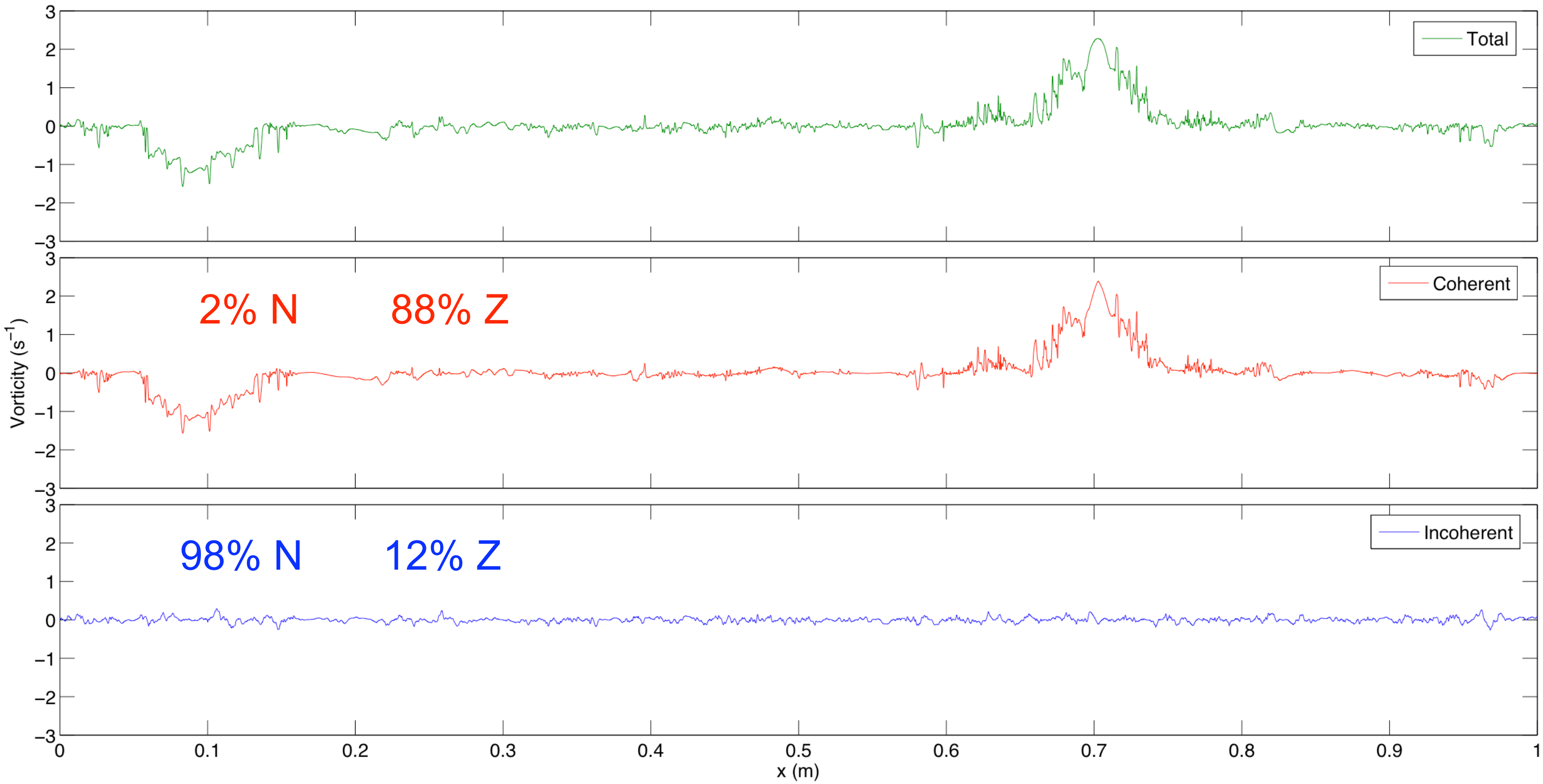
98% N
12% Z
0.01 % E

inc



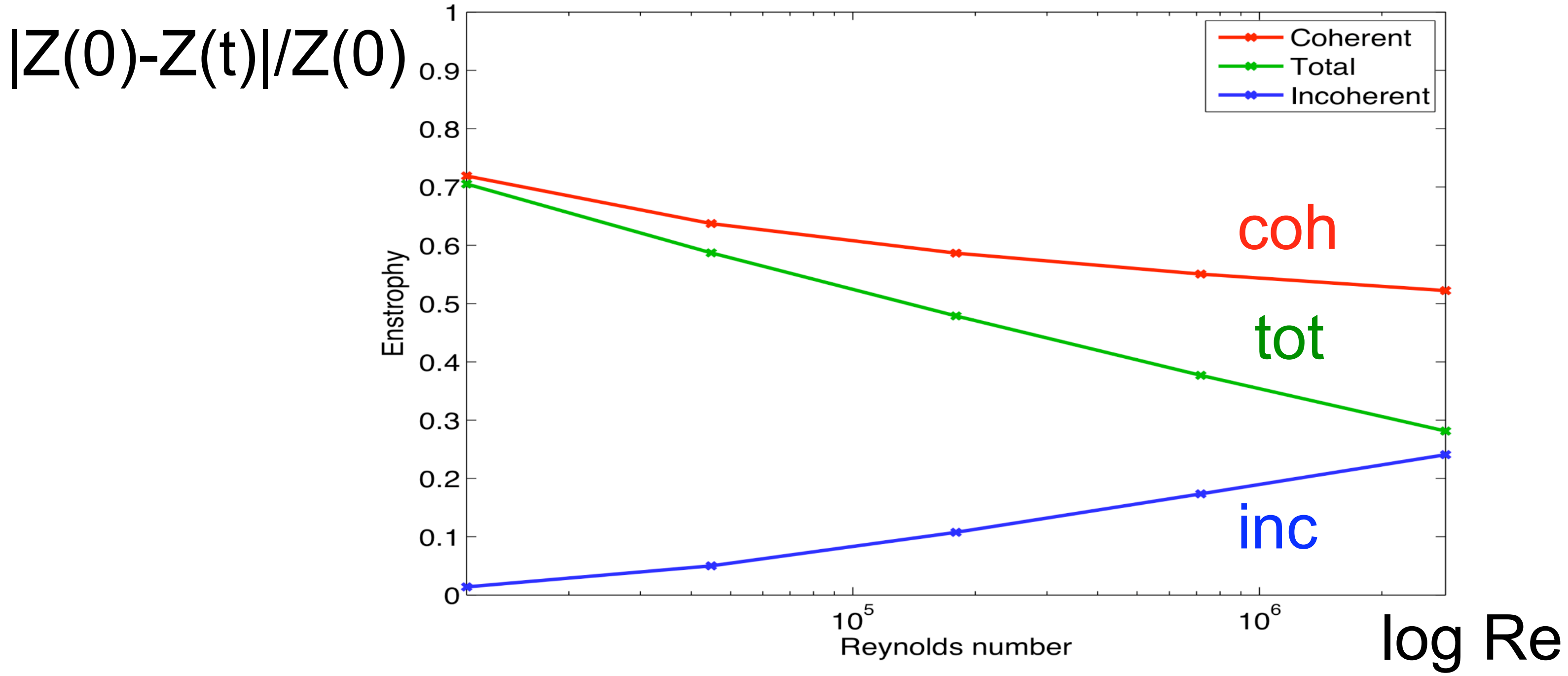
Coherent Vorticity Extraction

$N=8192^2$
$t=120\ \tau$



DNS
$N=8192^2$

Enstrophy dissipation



Wen Re tends to infinity, enstrophy dissipation vanishes as $(\ln Re)^{-1}$, as previously seen by Dmitruk and Montgomery (POF, 17, 2005) and by Tran and Dritschel (JFM, 559, 2006).
In contrast, coherent enstrophy dissipation tends to a constant.

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Comparison between
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and 3D Navier-Stokes
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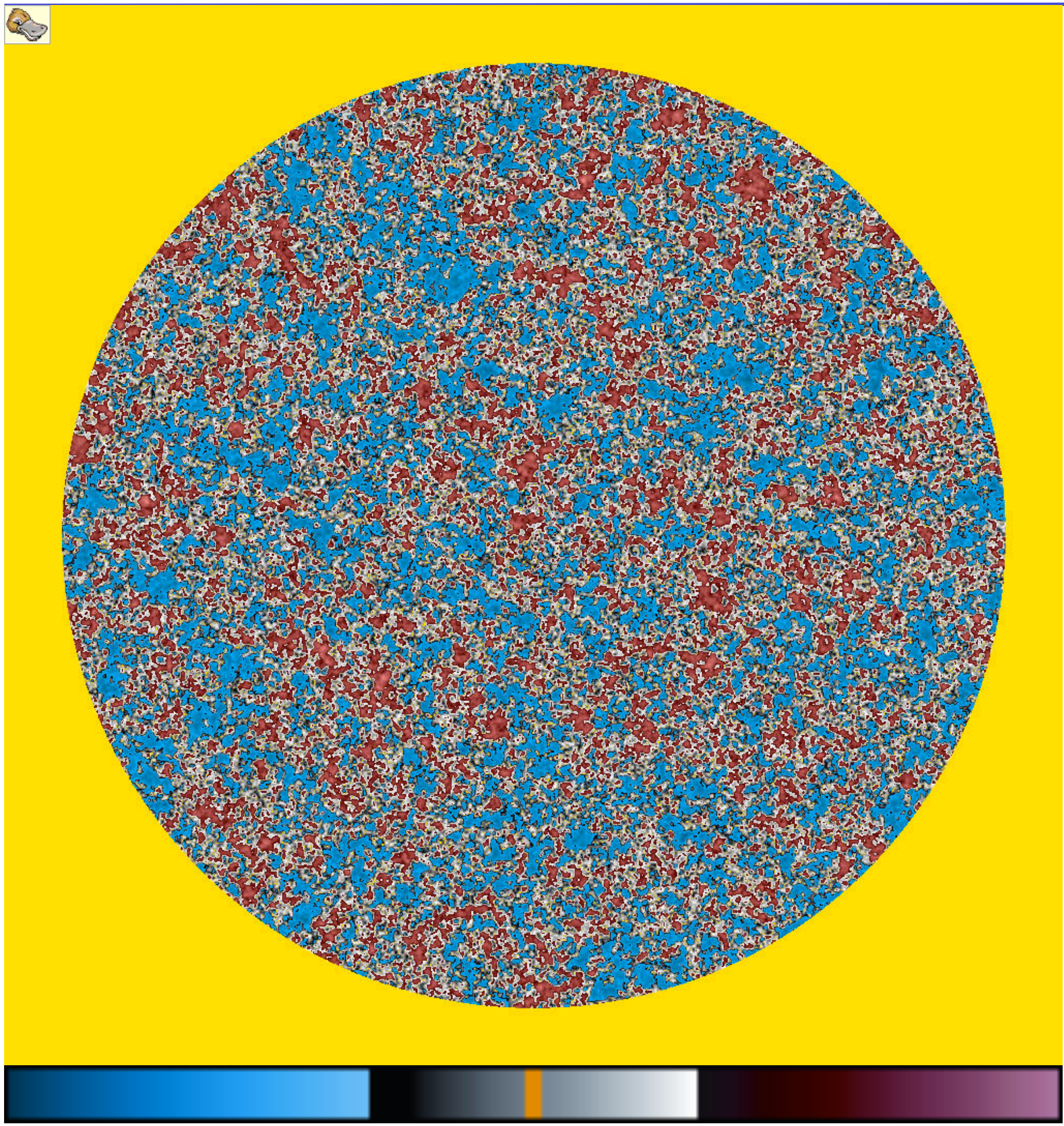
Wall-bounded 2D turbulent flow

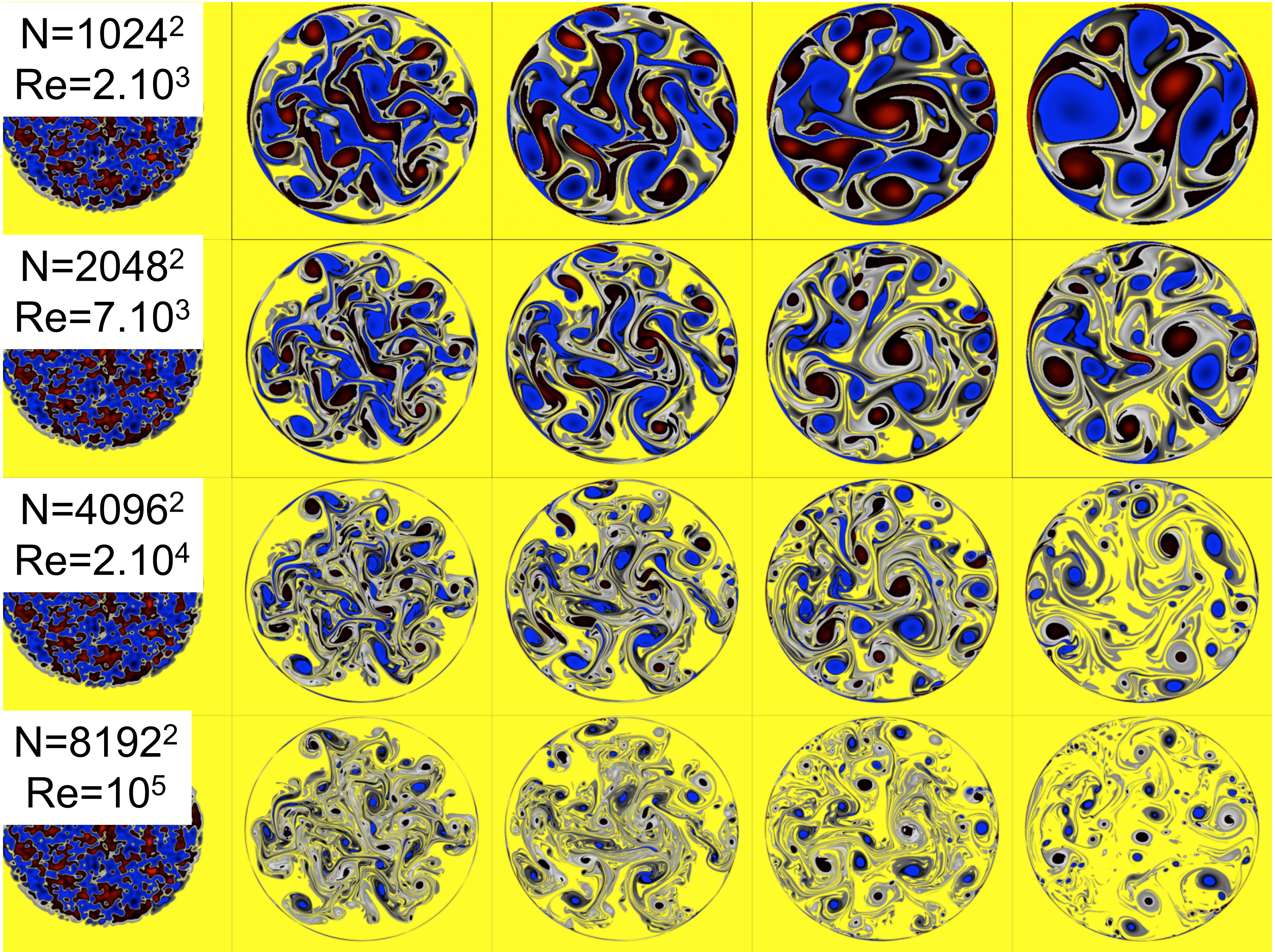


DNS
$N=1024^2$

Random initial conditions

No-slip boundary conditions using volume penalization

K. Schneider and M. Farge
Phys. Rev. Lett., ***95***, *244502, 2005*

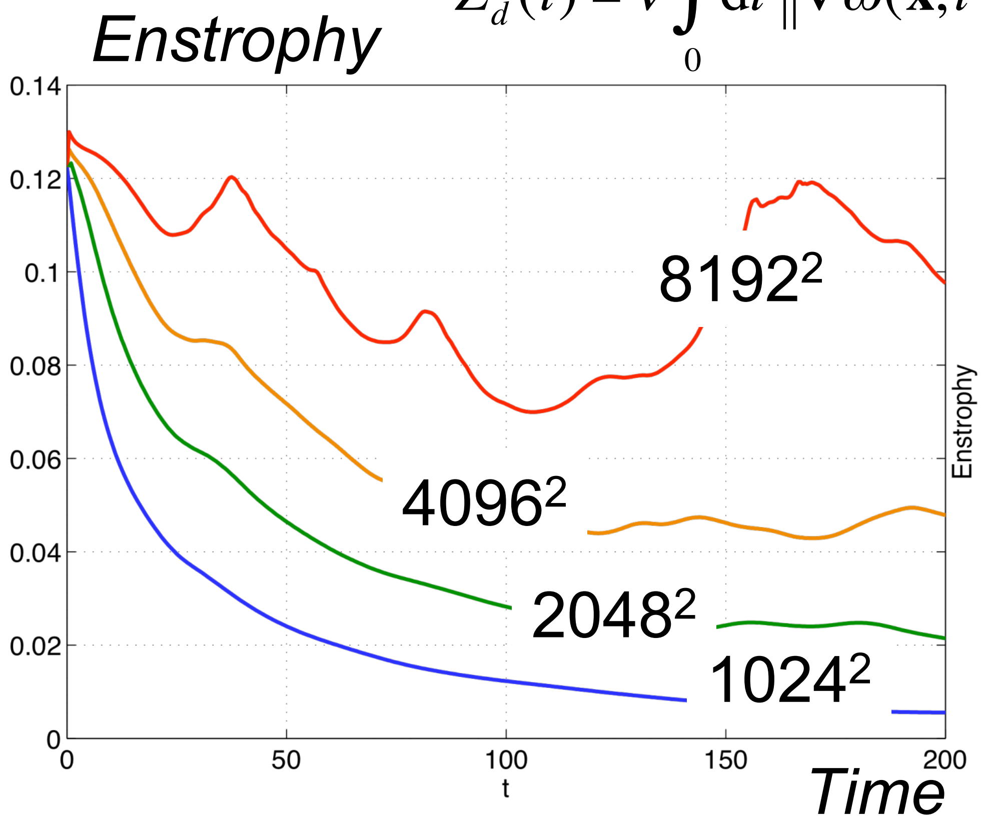


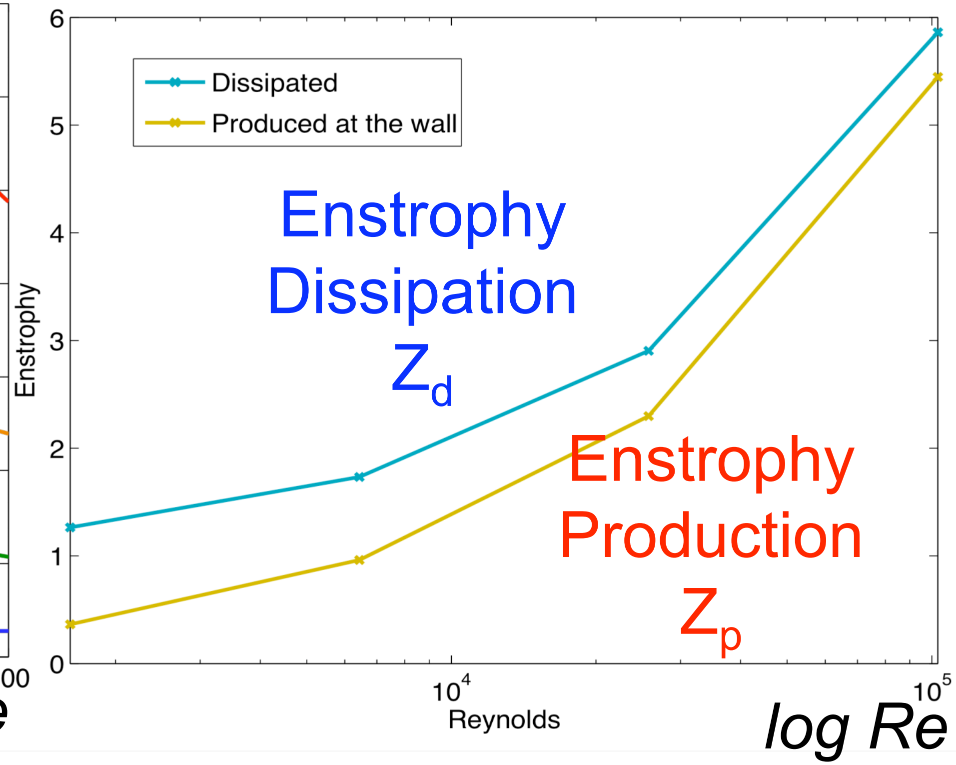
N=1024^2
Re=2.10^3
N=2048^2
Re=7.10^3
N=4096^2
Re=2.10^4
N=8192^2
Re=10^5

4 DNSs
from N=1024^2
to N=8102^2

2D viscous Navier-Stokes with no-slip walls

$$Z_d(t) = \nu \int_0^t \mathrm{d}t' \left\| \nabla \omega(\mathbf{x}, t') \right\|^2 \qquad Z_p(t) = Z(0) - Z(t) + Z_d(t)$$

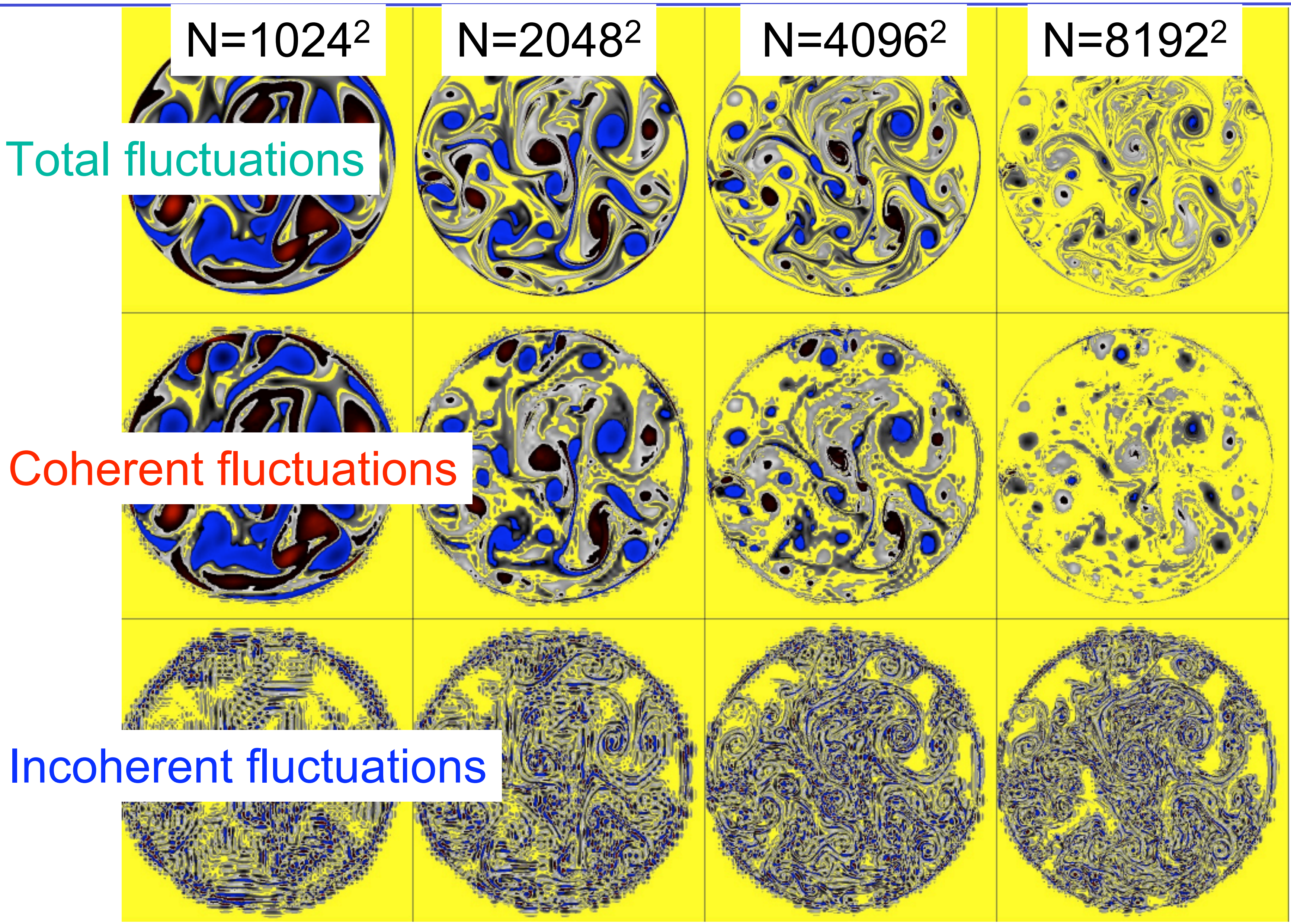




Enstrophy fluctuates more when Re increases since its production at the wall become more intermittent

Enstrophy production increases faster than enstrophy dissipation when Re increases

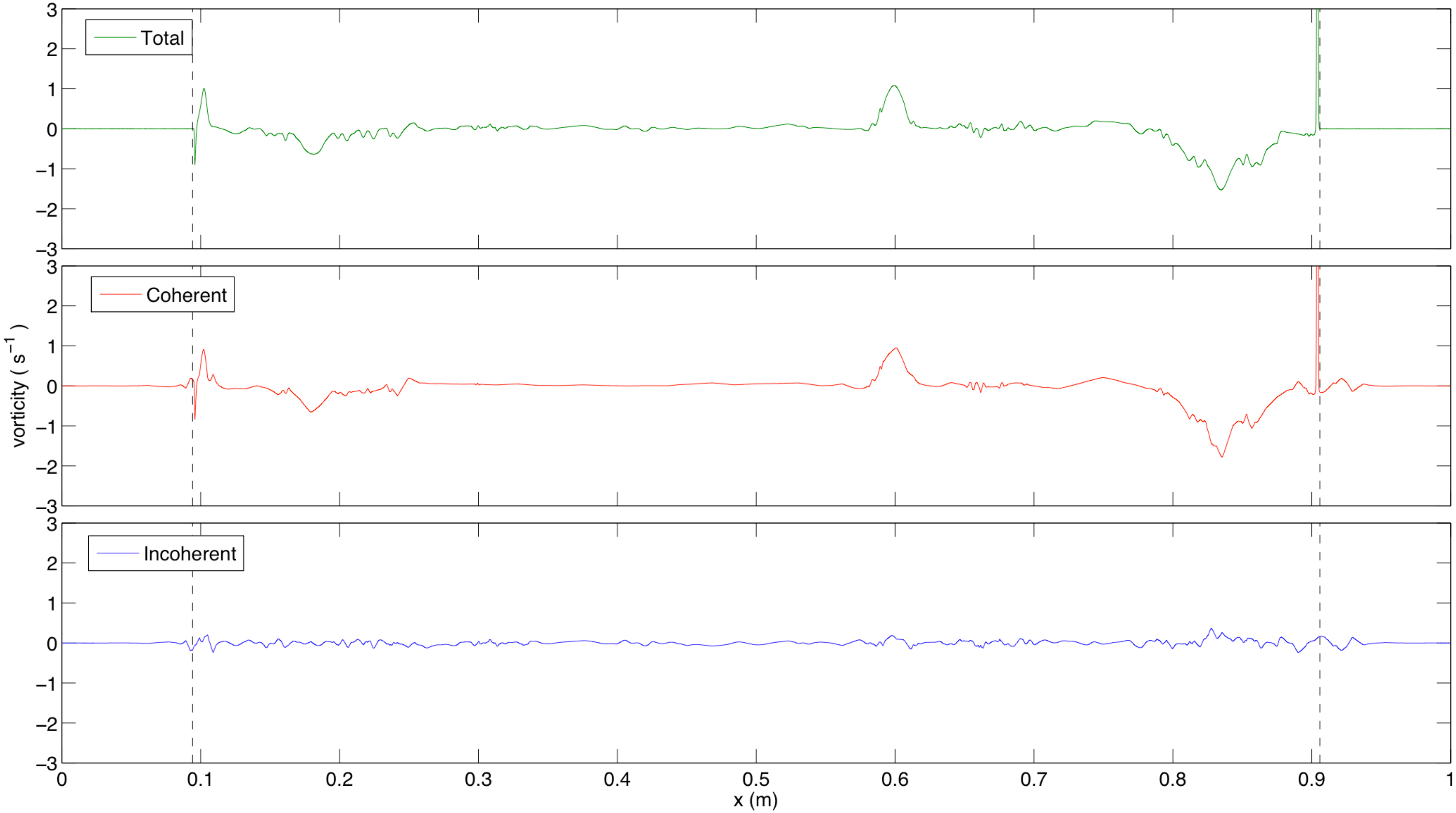
Coherent Vorticity Extraction



Coherent Vorticity Extraction

$N=8192^2$
$t=120\ \tau$

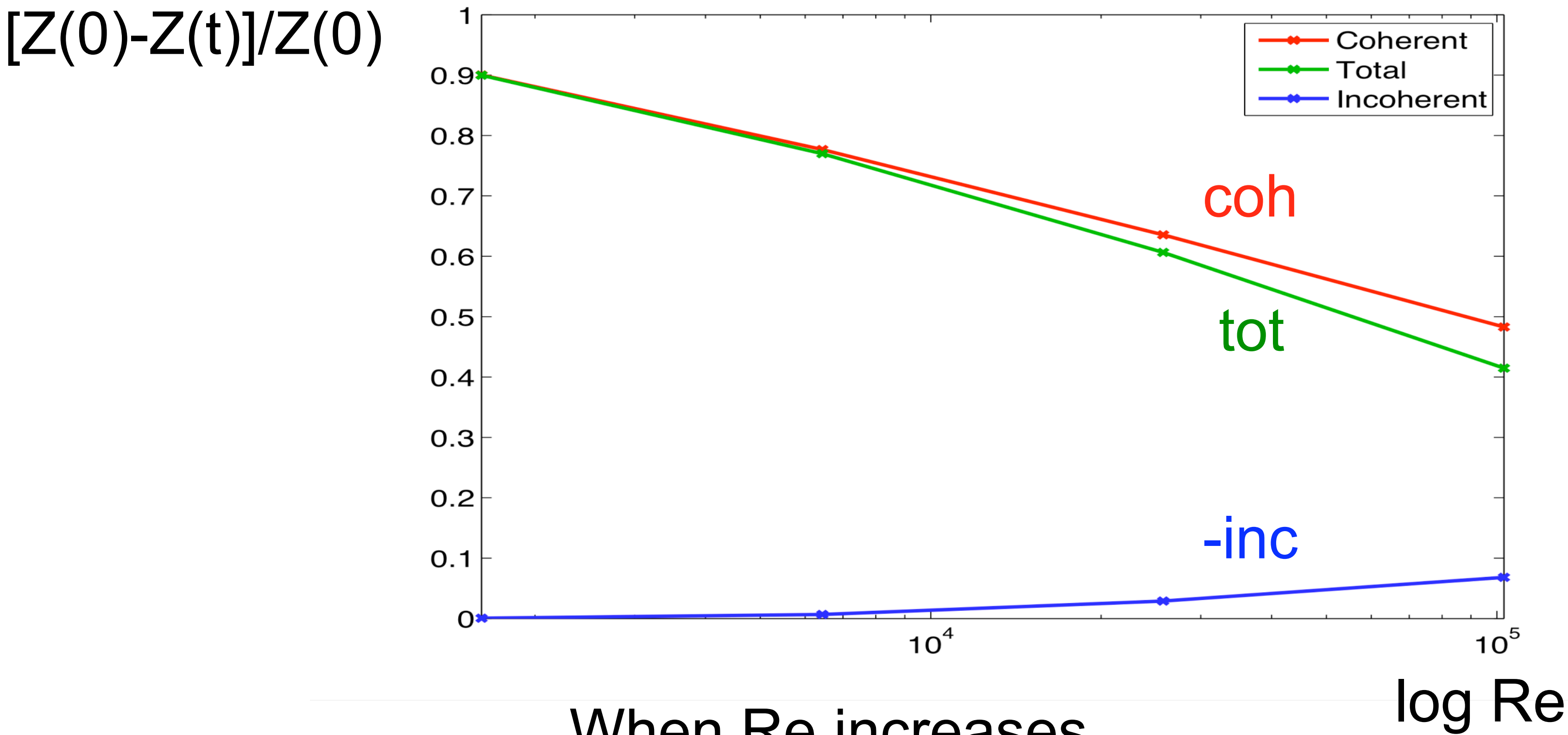
$\omega_{max}= 20$



DNS

$N=8192^2$

Enstrophy dissipation



When Re increases,

more and more coherent enstrophy is produced at the wall

and more and more incoherent enstrophy is dissipated.

Since production tends to overcome dissipation

we cannot yet evaluate how dissipation scales with Re.

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Comparison between
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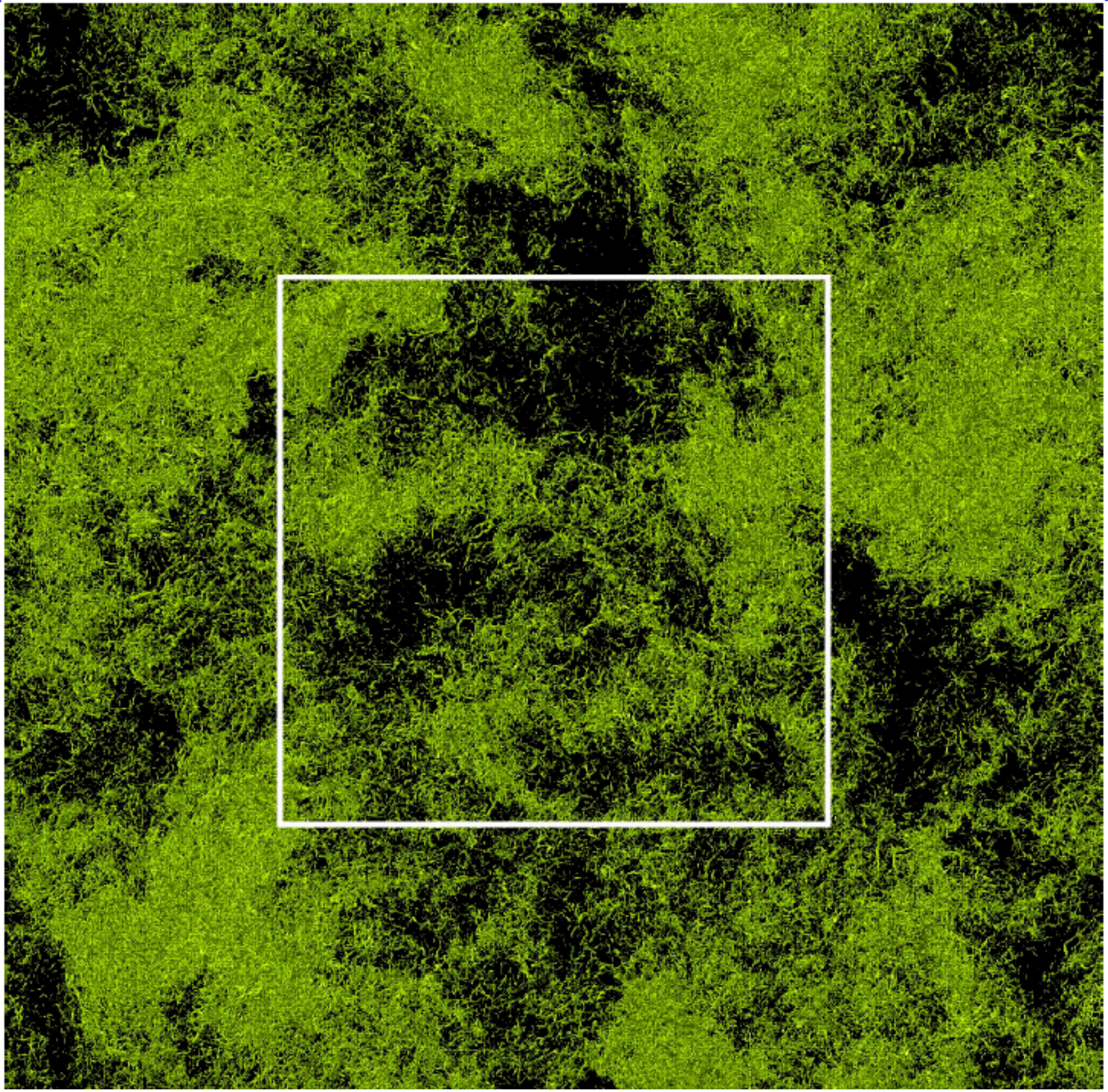
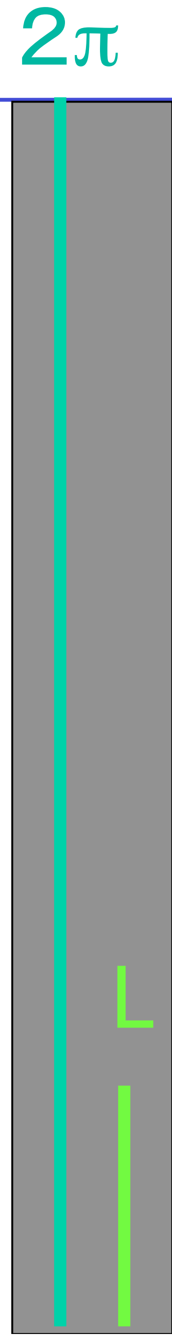
3D homogeneous isotropic turbulent flow

DNS
$N=2048^3$

L,
integral
scale

from
Yukio Kaneda
et al., 2002

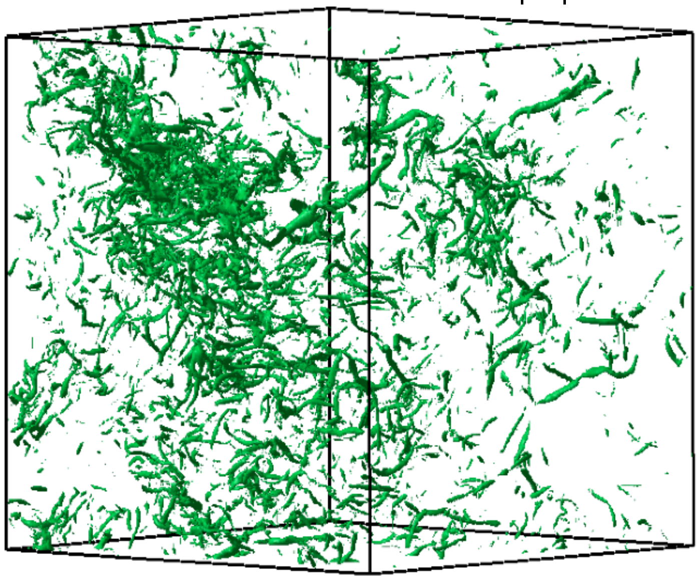




Coherent Vorticity Extraction

DNS
N=2048^3

$|\omega|=5\sigma$



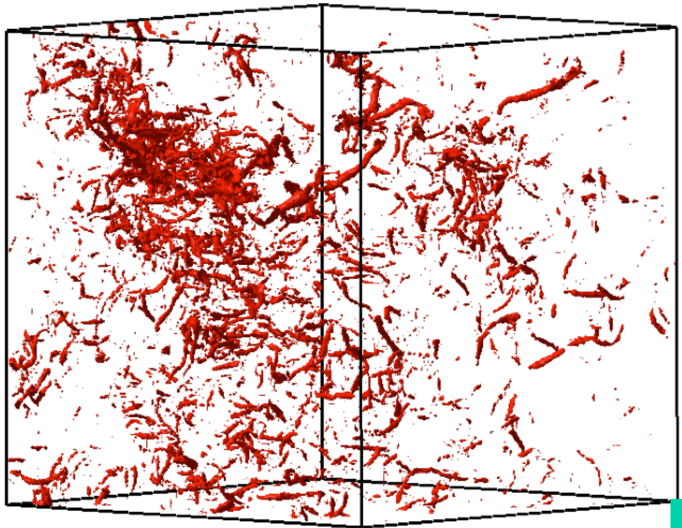
with $\sigma=(2Z)^{1/2}$

Coherent vorticity

2.6 % N coefficients
80% enstrophy
99% energy

Incoherent vorticity

97.4 % N coefficients
20 % enstrophy
1% energy

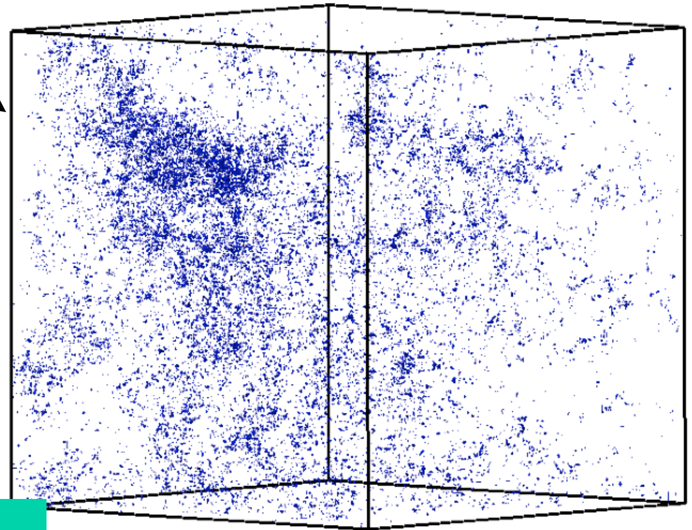


$|\omega|=5\sigma$

Total vorticity

R_λ=732
N=2048^3
Visualization
at 256^3

+

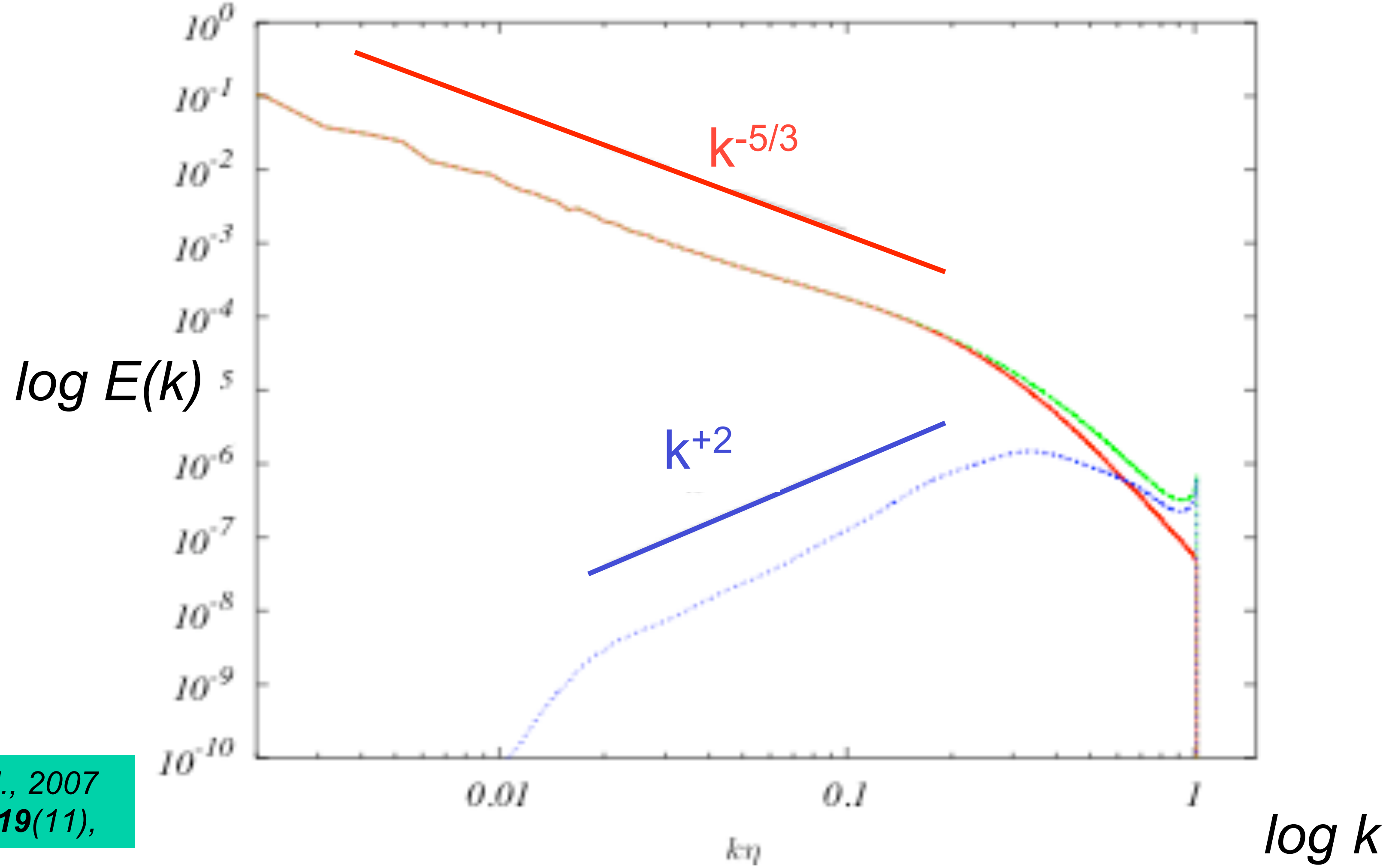


$|\omega|=5/3\sigma$

Okamoto et al., 2007
Phys. Fluids, ***19****(11)*

Energy spectrum

DNS
N=2048^3



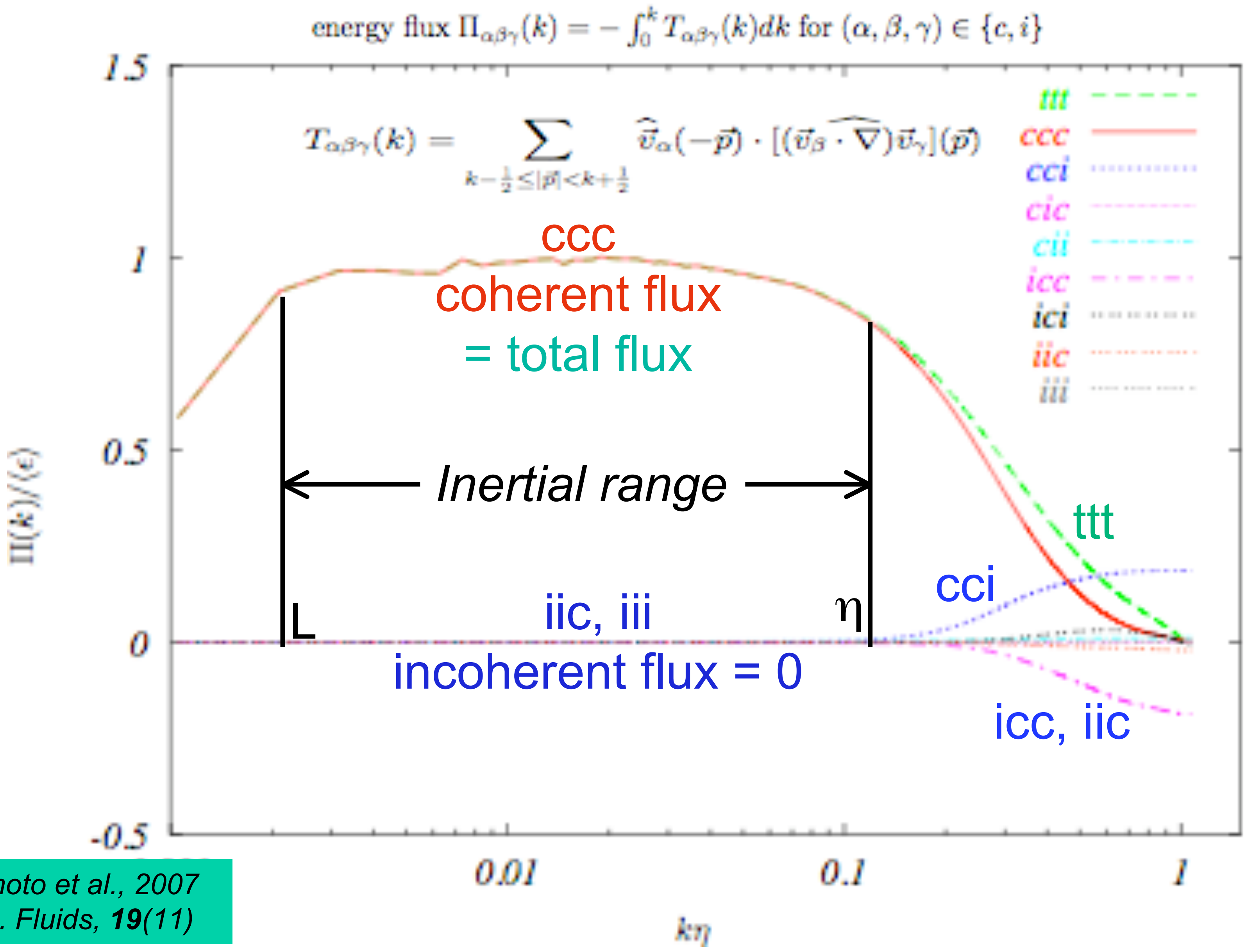
Okamoto et al., 2007
*Phys. Fluids, **19**(11),*

2.6 % N coefficients
80% enstrophy
99% energy

Multiscale Coherent
$k^{-5/3}$ scaling, i.e.
long-range correlation

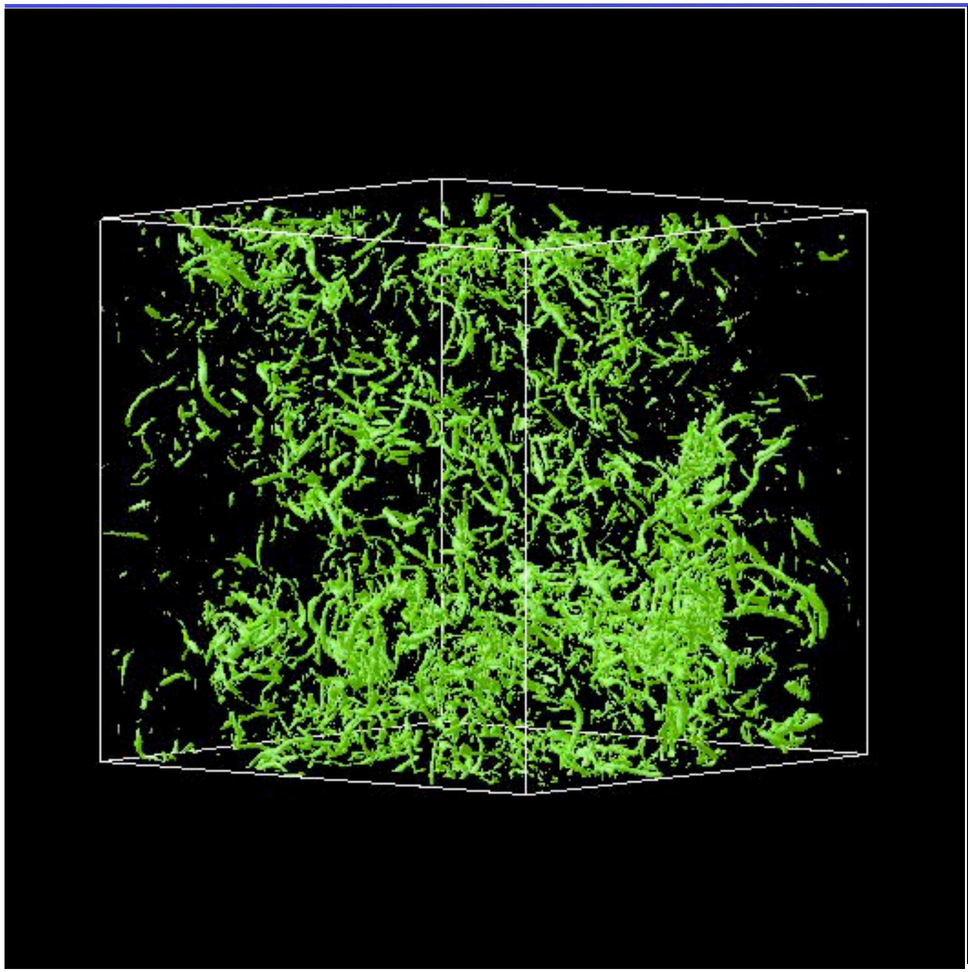
Multiscale Incoherent
k^{+2} scaling, i.e.
energy equipartition

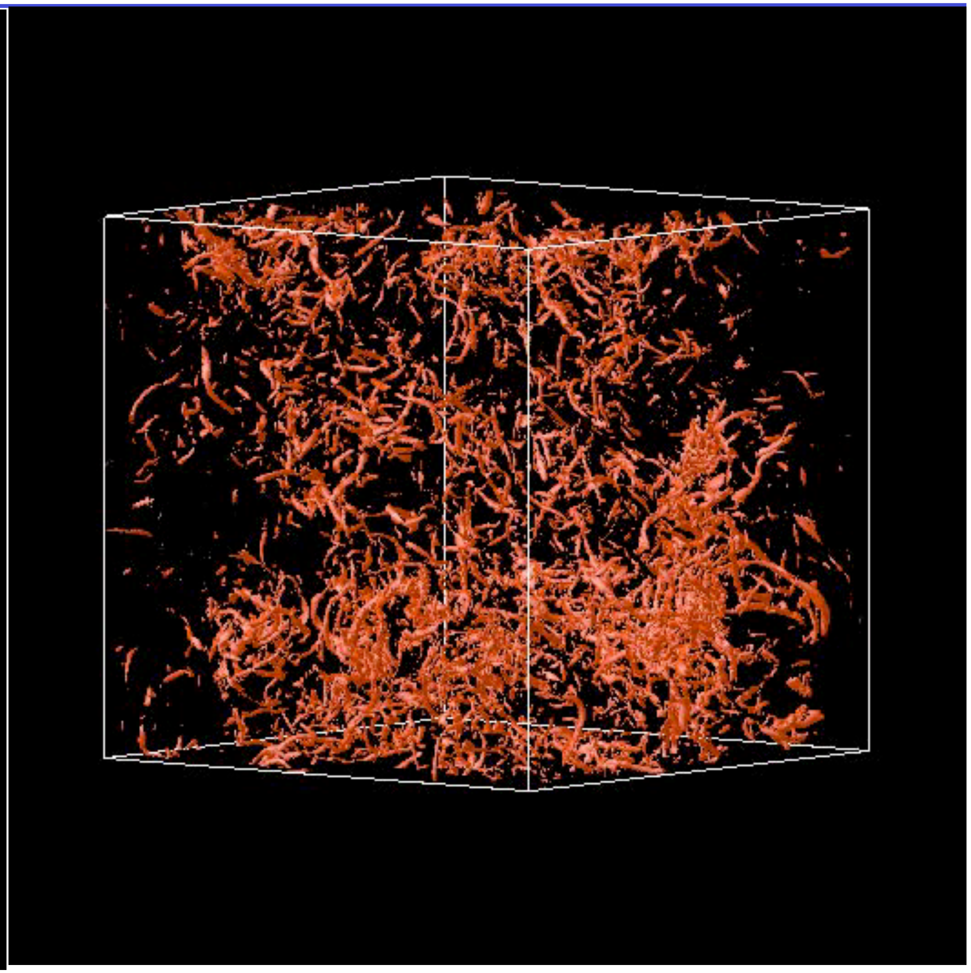
Nonlinear transfers and energy fluxes



Okamoto et al., 2007
Phys. Fluids, ***19****(11)*

3D decaying Navier-Stokes turbulence



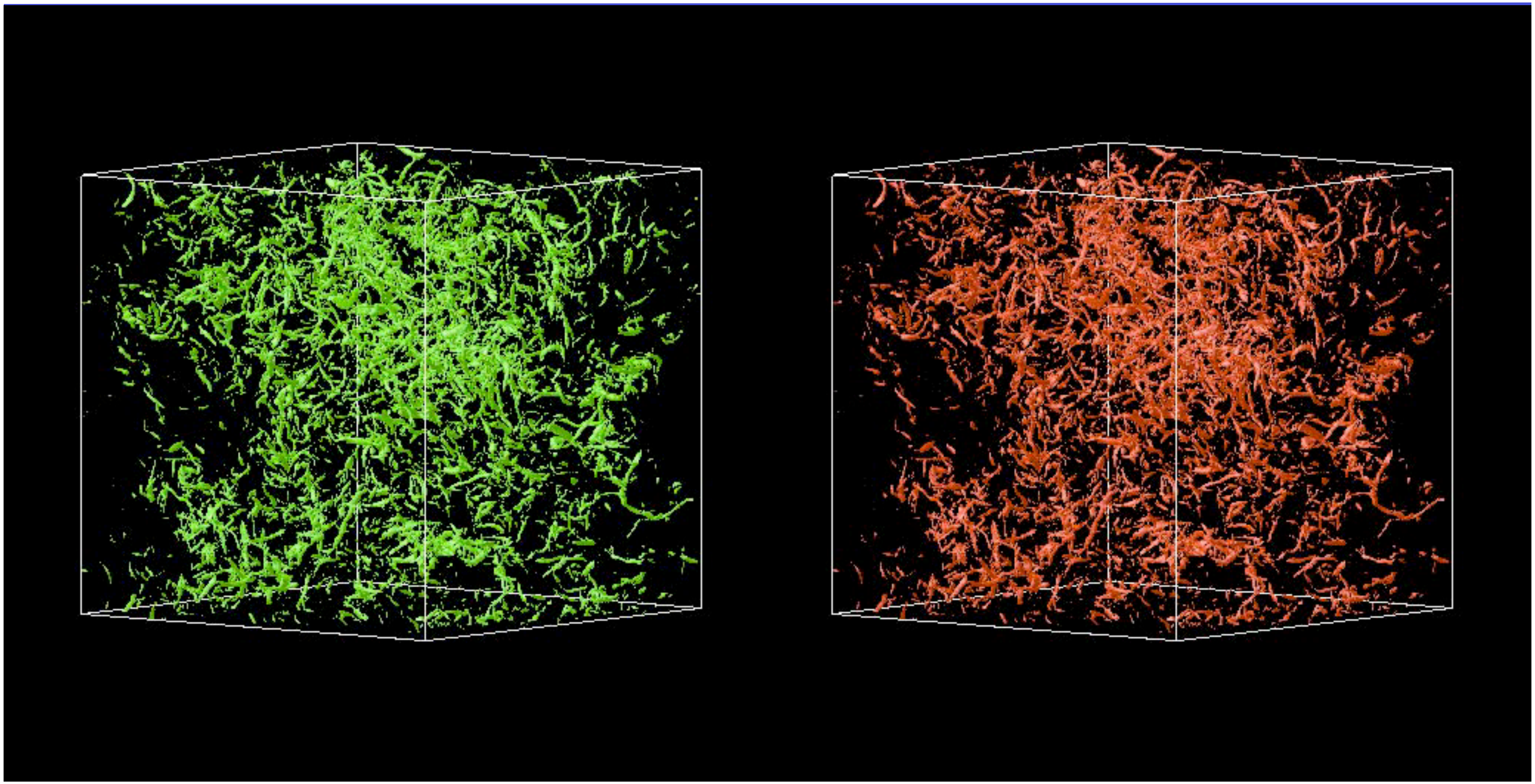


DNS
$N=256^3$

Okamoto, Yohsimatsu, Schneider, Farge and Kaneda, 2009, preprint

CVS
$N_c=(2\%+10\%)N$

CVS of 3D forced Navier-Stokes turbulence

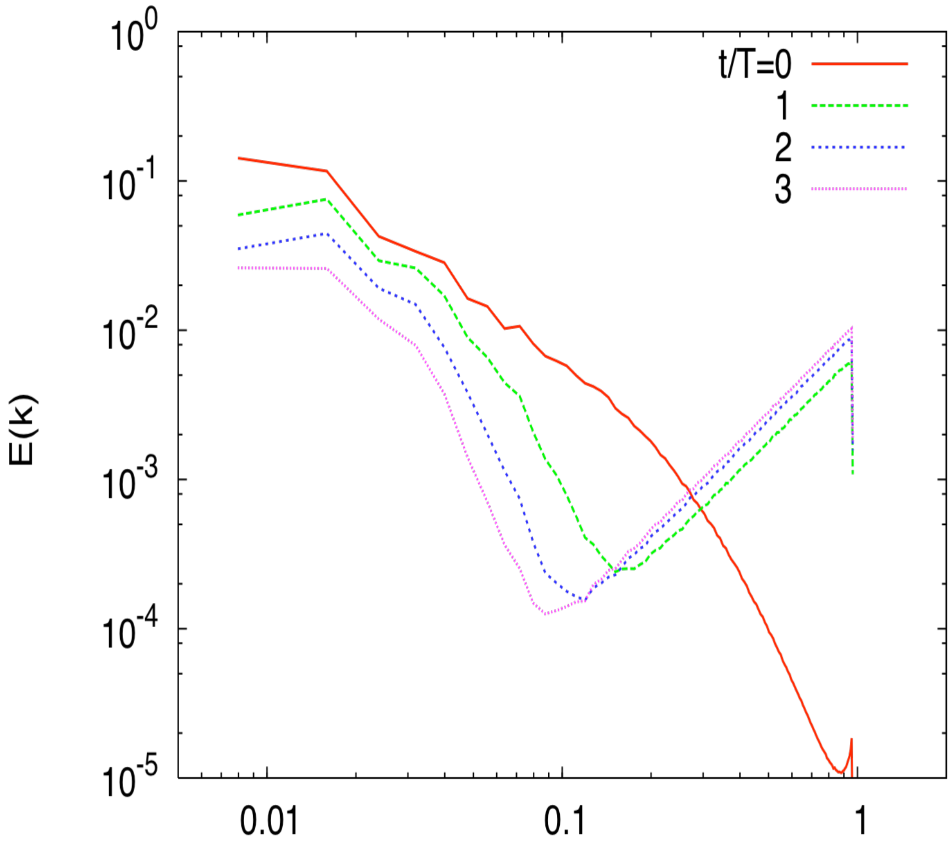


DNS
$N=256^3$

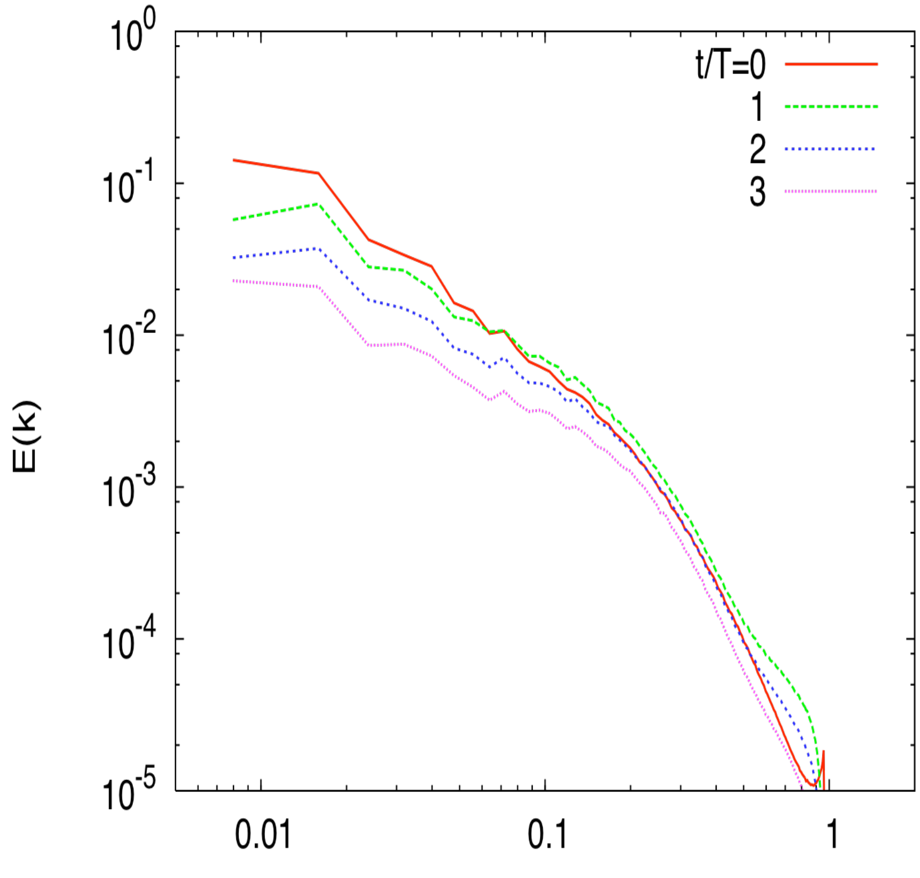
Okamoto, Yohsimatsu, Schneider, Farge and Kaneda, 2009, preprint

CVS
$N_c=(2\%+10\%)N$

CVS of 3D Euler turbulence



3D truncated Euler



3D CVS-Filtered Euler

DNS
$N=256^3$

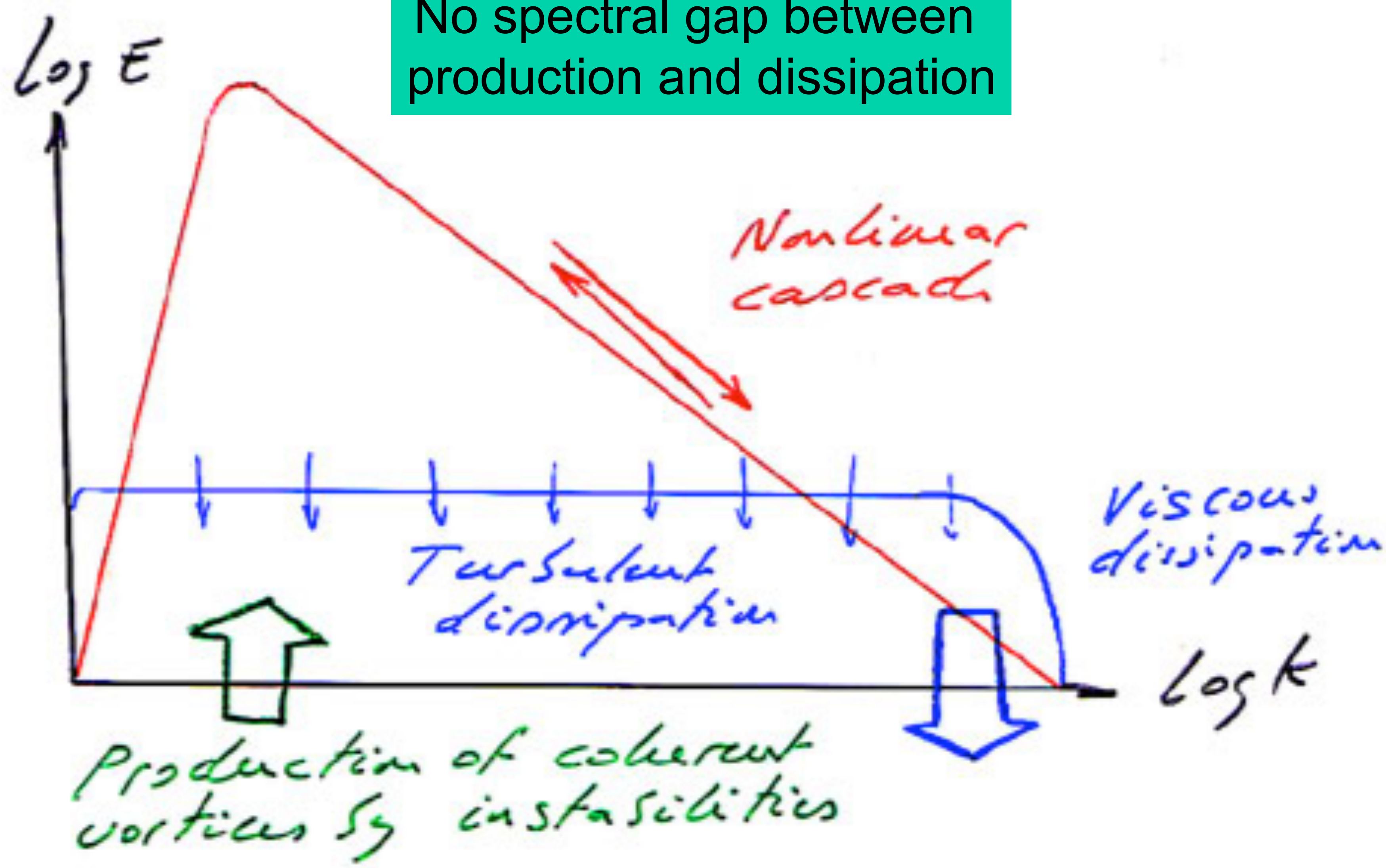
Okamoto, Yohsimatsu, Schneider, Farge and Kaneda, 2009, preprint

CVS
$N_c=(2\%+10\%)N$

New interpretation of the energy cascade

Fourier space viewpoint

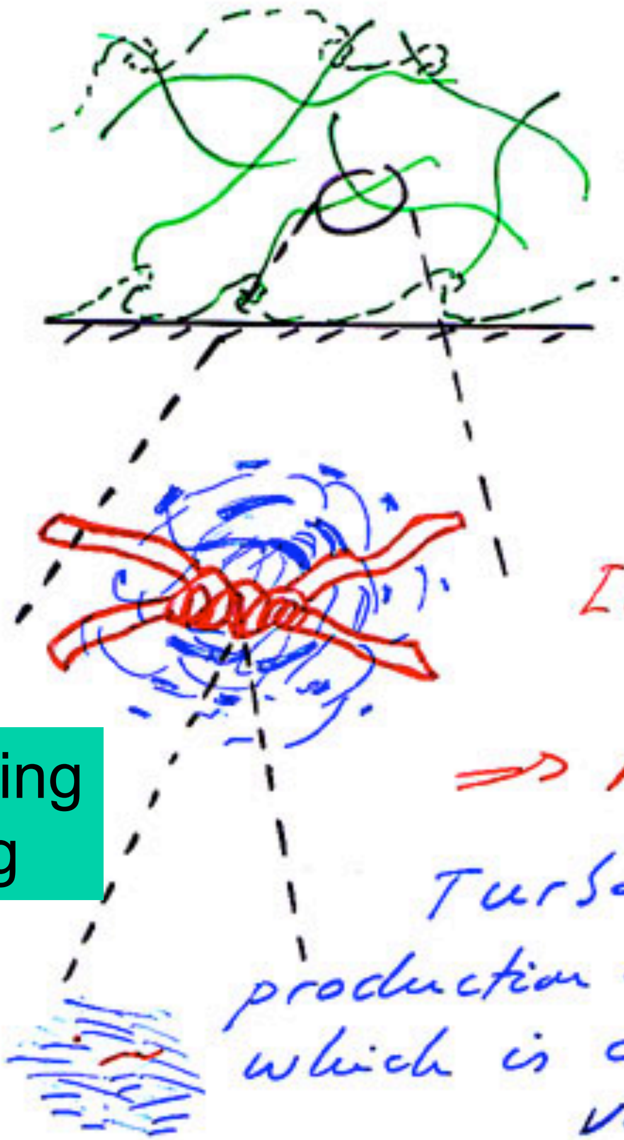
No spectral gap between production and dissipation



New interpretation of the energy cascade

Physical space viewpoint

No vortex fission 'à la Richardson'



PRODUCTION

Webs of vortex tubes produced by instabilities in shear layers

INERTIAL RANGE

Intermittent vortex interactions

⇒ Nonlinear cascade

+

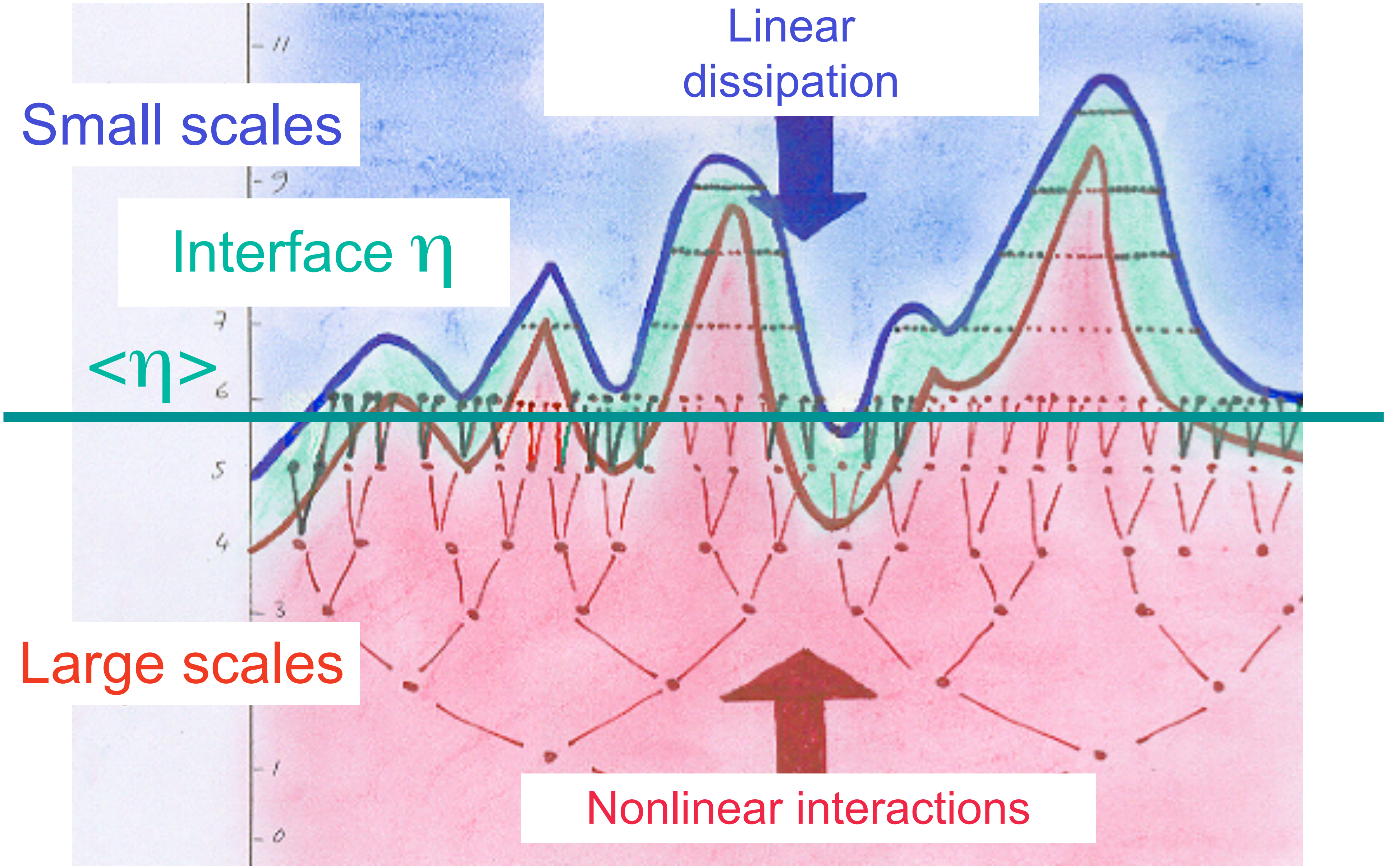
Turbulent dissipation

production of background noise which is damped in the viscous range

Vortex stretching and bursting

New interpretation of the energy cascade

Wavelet space viewpoint



Conclusion

We have shown that :

regularizing at each time step

the **truncated incompressible Euler** solution (ν=o)

with the CVS filter is equivalent to computing

the **incompressible Navier-stokes** solution (ν>o).

This gives evidence that :

retaining only **the strong wavelet coefficients** of vorticity

and reconstructing from them the induced velocity

preserves the **inviscid nonlinear dynamics**

We conjecture that :

discarding the **weak wavelet coefficients** of vorticity

may be sufficient to model **turbulent dissipation**.

http://wavelets.ens.fr

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