Usage Report for Fiscal Year 2020

Project Title:

Studying properties of many-body open quantum systems

Name:
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1. Background and purpose of the project, relationship of the project with other projects

Symmetries play a fundamental role in physics. Indeed, they constraint the equation of motion of a system, preventing or enhancing the emergence of collective behaviors. As such, symmetries are a fundamental tool to correctly capture the onset of criticality. Symmetries, and symmetry breaking, can distinguish phases of matter.

The driven-dissipative physics of light is the focus of intense research, fostered by the achievements of nonnegligible photon-photon interactions and sizeable light-matter couplings. These systems are driven out of their thermal equilibrium and do not obey the paradigms of thermodynamics. Even in the absence of free-energy analysis, open quantum systems display critical phenomena, such as dissipative phase transitions. Thus, the role of symmetries in open quantum system as attracted much interest [1]. In particular, the study of second-order phase transition in the presence of symmetries has proved effective [2].

Dissipative phase transitions are not the only critical phenomenon occurring in open quantum systems.
While a dissipative phase transitions is associated with the emergence of multiple steady states, a boundary time crystal is formed when permanent oscillations arise spontaneously in an otherwise time translation invariant system [3].

Both these critical phenomena can be understood using a Lindblad master equation and the corresponding Liouvillian superoperator, i.e., the matrix describing the evolution of the density matrix written in its vectorized form. A dissipative phase transitions emerges when one (or more) Liouvillian eigenvalues become zero in both real and imaginary parts. Similarly, for boundary time crystals, the Liouvillian acquires eigenvalues with zero real part but nonzero imaginary one.

In this FY, we characterized the properties of highly-symmetrical [$U(1)$] systems. On the one hand, we investigated the correspondence between boundary time crystals and phase transition. On the other, we studied within a purely quantum and dissipative description the Scully-Lamb laser model, unveiling novel properties of the lasing transition which we call a Liouvillian spectral collapse.

2. Specific usage status of the system and calculation method

We used diagonalization techniques for the study of the Liouvillian superoperator written in its matrix form. The Liouvillian we considered in these projects where large matrices (up size 1.000.000) making it impossible to store them on local machine with 16 Gb of RAM. We wrote an algorithm to transform these matrices in their block-diagonal form, in order to be able to simplify the computational task. Within this FY, we mainly used Hokusai supercomputer this task of exact diagonalization, and produced very interesting results (presented in the next section). Although we have finished this round of simulations, we still in the referee process for what it concerns the article on time crystal, and we about to submit the article on the the Liouvillian spectral collapse. As such, more simulation may be needed during the referral process.

３. Result

To efficiently diagonalize the Liouvillian, we introduce the following algorithm [c.f. Fig.1]:

- A matrix M is written as the adjacency matrix of an undirected graph;
- Each block in the block diagonal form is a single connected component in the graph; thus, the problem boils down to finding each connected component in the graph.
- We then use the Breadth First/Depth First search algorithm ¥cite{MooreBFS1959} consecutively to obtain the permutation matrices and the indices of the blocks. The time to perform this task (i.e., its computational complexity) is linear in the number of nodes in the graph.
- We use the permutation matrix to produce each block of M

Using this novel algorithm, we investigated criticality in open quantum systems. Concerning the time crystal-dissipative phase transition correspondence, we have demonstrated that, by a change of reference, it is possible to recast the formation of boundary time crystals in terms of breaking of $U(1)$ symmetries in a nonlinear optical model, where the nonlinearity comes from a dissipative term.

We have shown that, in the thermodynamic limit, a second-order dissipative phase transition in the rotating frame corresponds to a boundary time crystal in the laboratory frame. The two phenomena are the same in terms of the Liouvillian spectrum (or its gap) but just presented in different representations (frames). We find this prediction is the most important result of the manuscript. Some of the results are shown in Fig. 2 and Fig. 3, where we plot the photon number and the Liouvillian gap for different sizes of the system (proving the spontaneous symmetry breaking) and the two-time correlation functions of the system, displaying long-lasting oscillations signaling the eventual emergence of a time crystal in thermodynamic limit.

Concerning the Scully-Lamb laser model, we analyzed the lasing transition within the open quantum system framework provided by the spectral properties of the Liouvillian superoperator. Our analysis takes into account the non-equilibrium character of lasing and its quantum fluctuations, making this analysis more rigorous than previous studies, where the Landau theory and semiclassical analysis have been used. Within the Liouvillian formalism, we demonstrated that the stimulated emission in the Scully-Lamb laser model is associated with a spectral collapse. That is, a high degeneracy of the Liouvillian spectrum de-scribe diabolic points of infinite order, one for each symmetry sectors, triggering hysteresis and other critical properties which cannot be explained by the symmetry breaking alone.

By considering a generalized SLLM, i.e., the standard SLLM model with constant additional dephasing, the $U(1)$ symmetry is maintained, but long-lived phase coherences are destroyed, thus preventing the emergence of a symmetry-broken phase. Nevertheless, the system is critical as witnessed, e.g., by the number of photons inside the cavity, which is affected by the intensity of the dephasing and shows the same discontinuity as in the standard SLLM. Despite the absence of a symmetry breaking, a second-order phase transition takes place. We conclude that the second-order transition at the lasing threshold originates from the more fundamental structure of the spectral collapse, and symmetry breaking is a consequence and not the cause of the spectral collapse.

This criticality, even in the absence of spontaneous symmetry breaking, can be witnessed by the presence of a dynamical hysteresis. Indeed, hysteresis is absent in more standard second-order phase transitions. Remarkably, a semiclassical analysis, which neglects field fluctuations, completely misses this phenomenon. We find this quantum-fluctuation-induced multistability quite anomalous since, usually, multistability is suppressed by the presence of quantum fluctuations. For example, the semiclassical bistable solutions of a single Kerr resonator become a single density matrix, once quantum fluctuations are taken into account. This can be explained by the presence of a quantum tunneling time between the different semiclassical solutions, and true bistability (associated to a first order phase transition of the Kerr resonator) is only restored in the thermodynamic limit, where fluctuations are suppressed.

4. Conclusion

Our works have clarified the role of symmetries in open quantum system, allowing to connect apparently different phenomena within the same formalism. Furthermore, by applying our methods to the well-known Scully-Lamb model, we have been able to correctly characterize criticality in such an open and quantum systems, unveiling novel properties which previous studies of this transition missed.

The article on time crystal is under revision in this moment, and we plan to sumbit the one on the lasing transition in the next month.

5. Schedule and prospect for the future

We plan to proceed in two parallel directions: (i) The methods we developed to study these highly-symmetrical systems are of widespread interest. Thus, we plan to publish a paper in which we detail them. (ii) Having focused on single-cavity systems, we plan to extend our results to lattice geometry. Particularly interesting, is also the question of how dephasing acts on more "standard" phase transition, such as thermal or quantum phase transition. These questions can be efficiently studied using our algorithm.

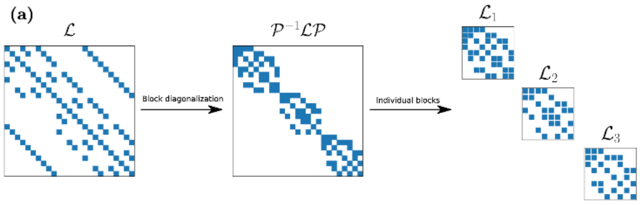

Fig. 1: Block diagonalization algorithm for any Liouvillian (a): Pictorial representation of the algorithm on a $\mathbb{Z}3$-symmetric Liouvillian. One can reshape the Liouvillian into its block-diagonal form. Each block can be then separated and diagonalized.

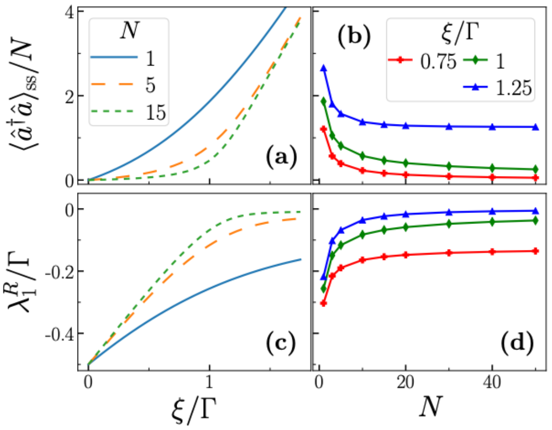

Fig. 2: Onset of the dissipative phase transition in the rotating frame. (a) Steady-state rescaled number of photons versus the incoherent drive strength ξ for different values of the rescaling parameter N (the thermodynamic limit is $N \rightarrow \infty$). (b) As a function of N for different ξ, rescaled photon number, showing the presence of two different regimes. (c) Liouvillian gap (in units of the damping rate Γ) versus ξ/Γ for different values of N, proving the closure of the Liouvillian gap (and thus the presence of a broken symmetry phase). (d) Liouvillian gap scaling as a function of N.

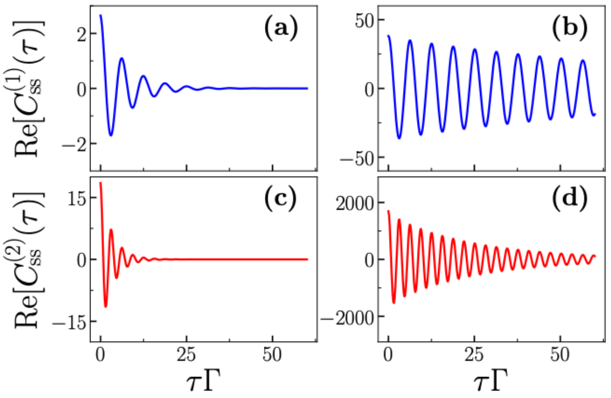

Fig. 3: Time crystal behavior witnessed by two-time correlation functions. Before the transition [(a) and (c)] oscillations are quickly suppressed, while after the transition [(b) and (d)] they last much longer, becoming infinitely lived in the thermodynamic limit. The point where these oscillations emerge coincides exactly with the spontaneous symmetry breaking.

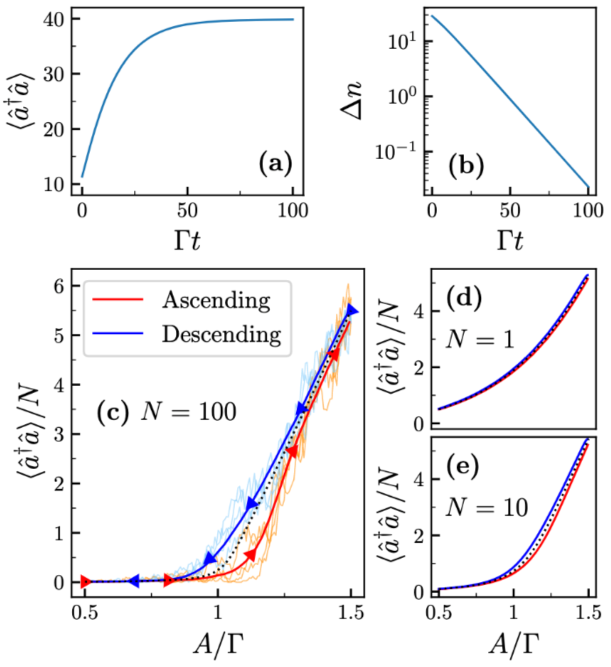

Fig. 4: As an example of the unusual phenomena we discovered in the Scully-Lamb model described within a Liouvillian theory, here we plot the presence of a dynamical hysteresis even in the absence of spontaneous symmetry breaking. While in Fig. (a) and (b) we show the presence of a divergently long timescale for the evolution of the photon number of the laser, in (c-e) we demonstrate that, by reaching the thermodynamic limit, the system criticality (even in the observe of symmetry breaking) allows to observe a critical phenomenon as the dynamical hysteresis.

[1] V. V. Albert and L. Jiang, *Symmetries and conserved quantities in Lindblad master equations*, Phys. Rev. A 89, 022118 (2014).

[2] F. Minganti, A. Biella, N. Bartolo, and C. Ciuti, *Spectral theory of Liouvillians for dissipative phase transitions*, Phys. Rev. A 98, 042118 (2018).

[3] F. Iemini, A. Russomanno, J. Keeling, M. Schiro,
M. Dalmonte, and R. Fazio, *Boundary Time Crystals*,
Phys. Rev. Lett. 121, 035301 (2018)

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[Paper accepted by a journal]

None

[Conference Proceedings]

None

[Oral presentation]

None

[Poster presentation]

None

[Others (Book, Press release, etc.)]

arXiv preprint: Fabrizio Minganti, Ievgen I. Arkhipov, Adam Miranowicz, and Franco Nori, *Correspondence between dissipative phase transitions of light and time crystals*, arXiv:2008.08075