

**Project Title: Transport properties of self-propelled micro-swimmers**

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**I. Background And Purpose:**

Self-propelling Janus particles (JPs), the most common class of artificial microswimmer, have been the focus of widespread attention over the last two decades due to their emerging applications in nano-technology and medical sciences [1-3]. Such particles are made by coating one hemisphere with catalytic or photo-sensitive or magnetic materials. Under appropriate conditions, one hemisphere undergoes physical or chemical changes with respect to the other, thus producing some local gradient in the suspension fluid (self-phoresis). This strategy allows artificial swimmers to propel themselves by harvesting energy from their environment.

Thanks to their self-propulsion mechanism and in contrast to their passive peer, artificial swimmers can diffuse orders of magnitude faster and are capable of performing autonomous motion in periodic structures with broken spatial symmetry and exhibit other peculiar transport properties. Inspired by these unique transport features, researchers aim to design customized JPs to be used, for instance, as "nano-robots" capable of performing accurate mechanical operations. Additional promising technological applications have also been proposed. Among the most appealing ideas being pursued, we mention here the recent attempt to power passive particles through the self-propulsion mechanism of intermediary active particles to be used as controllable stirrers.

Adding a relatively small fraction of highly motile microswimmers to a suspension of less motile microswimmers can considerably enhance the overall motility of the mixture due to the stirring effect. This effect easily can be demonstrated in the

presence of inertia. But what will happen to overdamped or mass-less particles? This limit corresponds to low Reynolds numbers, a hydrodynamic regime that applies to most microswimmers investigated in the literature, both biological and artificial. This raises a problem, because, the velocity distribution of massless particles is mathematically ill-defined. Thus, the host-guest motility transfer due to the stirring effect cannot be perceived in the overdamped regime. To avoid this difficulty, in our simulations we computed an alternative motility quantifier in the overdamped limit, namely the effusion rate of the active JPs through a narrow pore of the simulation box.

Mixtures of interacting active particles (either of the same or different kinds) behave quite differently in many ways. For dilute solutions, particles interact via long-range hydrodynamic flows generated by active particles and the short-range interactions can be safely ignored. However, transport properties of dense mixtures are mostly dominated by the short-range interactions, which are responsible for a variety of cluster and pattern formation processes reported in the recent literature.

Our simulation of binary active mixtures shows that adding a fraction of active microswimmers, such as self-propelled JPs, to a suspension of passive colloidal particles, results in a motility increase of the latter species. However, adding a small fraction of more active particles to a suspension of less active microswimmers, results in a non-trivial behavior, whereby the added species appears to enhance the motility of the host species. Such a mechanism can be controlled by tuning the parameters of the guest species, e.g., the intensity of light in the case of

light-induced JP using laser beam, near-infrared light or visible light.

Note that in an previous occasion (fiscal year 2017) we worked on the motility transfer in binary mixture that focus on the particles with inertia (that is stirring effect works there). Simulation results in presented this report (produced in fiscal year 2019) explore motility transfer of highly damped particles, where stirring effect does not work and motility transfer is a non-trivial effects.

## 2.Model:

Let us consider a two-dimensional system consisting of two types of JPs with different self-propulsion speeds in a thermal bath:  $N_w$  with speed  $v_w$  and  $N_s$  with speed  $v_s$ , For very short distances they interact with each other via a truncated Lennard-Jones potential,

$$V_{ij} = 4\epsilon \left[ \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^6 \right], \text{ if } r_{ij} \leq r_m$$

$$= 0 \text{ otherwise,}$$

where  $r_m$  locates the potential minimum and  $\sigma = 2r_0$ . The corresponding Langevin equations, describing motion of the particles in the mixture in the highly damped situation are,

$$\dot{x}_i = \sum_j F_{ij}^x + v_0 \cos \phi_i + \sqrt{2D_0} \xi_i^x(t) \quad (1)$$

$$\dot{y}_i = \sum_j F_{ij}^y + v_0 \sin \phi_i + \sqrt{2D_0} \xi_i^y(t) \quad (1)$$

$$\dot{\theta}_i = \sqrt{2D_\theta} \xi_i^\theta(t). \quad (3)$$

The particle with instantaneous position  $\{x_i, y_i\}$  diffuses in the force field under the combined action of self-propulsion and equilibrium thermal fluctuations.  $\{\xi_i^x, \xi_i^y\}$  are components of random thermal fluctuation, responsible only for translational diffusion, is modeled by a Gaussian white noises with,

$$\langle \xi_i^q(t) \rangle = 0$$

$$\langle \xi_i^q(t) \xi_j^{q'}(0) \rangle = 2\delta_{ij} \delta_{qq'} \delta(t)$$

Where  $q$  or  $q' = \{x, y\}$ . The thermal noise strength,  $D_0$

$= k_B T / \gamma$  can be estimated by measuring the translational diffusion of a free JP in the bulk in the absence of propulsion.

The propulsion velocity  $v_i$ , with modulus  $v_0$ , directed making an angle  $\phi_i$  with respect to the laboratory  $x$ -axis. Due to rotational diffusion of the particle,  $\theta_i$  changes randomly, which can be described as a Wiener process, as described in Eq.(3), with following statistical properties

$$\langle \xi_\theta(t) \rangle = 0$$

$$\langle \xi_\theta(t) \xi_\theta(0) \rangle = 2\delta_{ij} \delta(t)$$

The rotational diffusion constant  $D_\theta$  is related to the viscosity ( $\eta_v$ ) of the medium, temperature ( $T$ ) and size of the particle. For spherical particle with radius  $a$ , rotational diffusion constant can be expressed as  $D_\theta = k_B T / 8\pi\eta_v r_0^3$ . However, rotational diffusion may contain contribution of gradient fluctuations that are very much related to mechanism of acquiring self-propulsion. The mechanisms and origins of the translational and rotational diffusion may not be the same. Thus,  $D_0$ ,  $v_0$ , and  $D_\theta$  can be treated as independent model parameters.

We numerically integrated Eqs. (1-3) using a standard Milstein algorithm to obtain effusion rates of the both mixture species. The numerical integration was performed using a very short time step,  $10^{-6}$ - $10^{-7}$  to ensure numerical stability. At  $t = 0$ , the particles were uniformly distributed in the box with random orientation. To keep the number density of both species constant, a particle of the same species was re-injected with random position and orientation inside the box, whenever one had escaped through the pore. The datapoints reported in the figures shown here, have been obtained by ensemble averaging over a minimum of 10,000 trajectories.

## Result and discussions:

The effusion rate has been studied in depth to characterize classical transport unconstrained geometries [4]. We define the effusion rate of the

*strong (s) [weak (w)]* JPs,  $E_s$  ( $E_w$ ), as the number of  $s$  ( $w$ ) particles exiting the simulation box per unit time. In the case of a single-component system, we denote the effusion rate by  $E_m$ .

Effusion of non-interacting species: Let us consider the effusion rate  $E_m(0)$  of a single species of non-interacting JPs. In Fig.1(a) we plotted a few curves  $E_m(0)$  versus  $v_0$  for different values of  $D_\theta$ . For  $v_0 \rightarrow 0$  the effusion is controlled by thermal motion and, as expected, is insensitive to  $v_0$ . Effects due to self-propulsion become appreciable only for values of  $v_0$  larger than the particles thermal speed. Beyond this critical value, the effusion rate grows first quadratically with  $v_0$  and then saturates toward an asymptotic value.

The rising branches occur for  $l_\theta > x_L, y_L$ . Indeed, for very short rotational relaxation times  $\tau_\theta$  when it can safely be assumed that particles diffuse in a thermal bath with effective constant  $D_{\text{eff}}$  the effusion rate through a narrow pore of effective width much smaller than the confining box.

$$E_i = \pi \rho_i D_{\text{eff}} \left[ \ln \left( \frac{x_L + y_L}{\Delta} \right) \right]^{-1}$$

Here, the suffix  $i$  refers to either  $s$  or  $w$ ,  $\rho_i$  denotes the number density of the mixture component  $i$ , and  $D_{\text{eff}}$  is now,  $D_0 + v^2_0/2D_\theta$  – see Eq.(5). This estimate for  $E_i(0)$  agrees fairly closely with the simulation results reported in Fig. 5(a). In the opposite rotational regime, when  $l_\theta \gg x_L, y_L$ , the slow direction changes of the self-propulsion velocity tends to suppress the particles effusion through the pore. Assuming that  $\tau_\theta$  is much larger than any other system time scale, the effusion rate can be approximated by,

$$E_i(0) \approx x_L y_L \rho_i D_\theta / \pi$$

This asymptotic estimate has been marked in Fig. 5(a) by horizontal arrows.

The effusion rate of interacting self-propelling particles with  $\varepsilon > 0$  is plotted in the inset of Fig. 5(a). This figure shows the  $v_0$ -dependence of the effusion

rate relative to the corresponding rate in the absence of interaction,  $E_m(\varepsilon) = E_m(0)$ , for several values of  $D_\theta$ . The system we simulated here was quite dense ( $\phi = 0.66$ ), so that the self-propulsion mechanism becomes strongly constrained, being  $l_s \ll l_\theta$ . Like in non-interacting systems, the effusion rate is insensitive to self-propulsion with low  $v_0$ . More remarkably, with increasing  $v_0$  the relative effusion decreases.

We attributed this result to the jamming of the interacting particles caused by self-propulsion near the box walls. Snapshots of the mixture configurations [Fig 2 and Fig.3(b) (inset)] corroborate this assertion. Jamming effect gets noticeable as soon as the self-propulsion length becomes larger than the confining box.

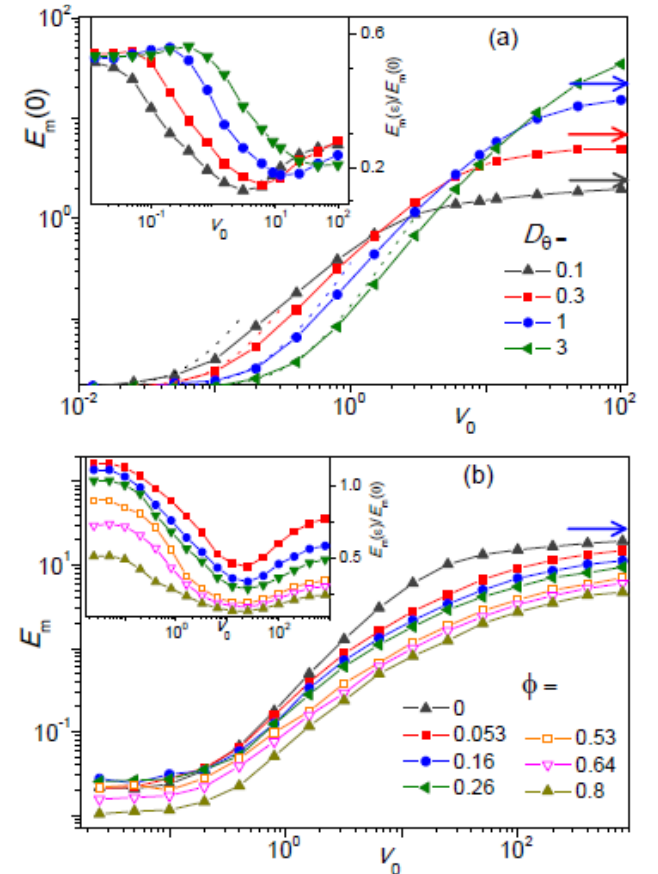


Fig1: (a) Effusion rate  $E_m(0)$  of non-interacting JPs with  $\varepsilon = 0$ , as a function of the self-propulsion speed,  $v_0$ , for different rotational diffusion coefficient  $D_\theta$ . Dotted lines are the predictions based on Eq. (16). Horizontal arrows indicate the corresponding rate upper bound, Eq. (17), for large  $\tau_\theta = 1/D_\theta$ . Inset: the effusion rate ratio  $E_m(\varepsilon)/E_m(0)$  for  $\varepsilon = 0.1$  and

different  $D_0$  (see legends). (b) Effusion rate  $E_m(\varepsilon)$  of interacting self-propelled particles versus  $v_0$  for  $e = 1$  and different packing fraction  $f$ . Inset:  $E_m(\varepsilon) = E_m(0)$  versus  $v_0$  for the same set of parameter as the main panel. Other simulation parameters for main panels and inset:  $x_L = y_L = 10$ ;  $\Delta = 0.5$ ;  $r_0 = 0.5$ ;  $D_0 = 0.03$ ;  $N_m = 80$ .

More remarkably, excluded volume effect become apparent for  $v_0 \rightarrow 0$ ; the interacting particles become more effusive than the non-interacting ones. In a dilute solution, this effect persists until self-propulsion length grows larger than the average inter-particle spacing. This explains why, in the presence of strong self-propulsion, the computed effusion ratios still grow with  $v_0$ , though quite slowly. Therefore, the appearance of such an effect and minima of  $E_m(\varepsilon)/E_m(0)$  versus  $v_0$  are inversely related to the rotational diffusion [see inset of Fig. 1(a)]. By the same token, one expects that both decaying and

are quite insensitive to the packing fraction,  $\phi$ , in agreement with the data plotted in the inset of Fig. 1(b). In the very strong self-propulsion regime, both  $E_m(\varepsilon)$  and  $E_m(0)$  tend to saturate [see Fig. 1(b)]. However,  $E_m(\varepsilon)$  saturates at larger  $v_0$  values than in the non-interacting case. A plausible explanation is suggested by a comparison of the mixture snapshots [see Fig. 2]. The particles far away from the walls are more mobile and contribute more to the effusion rate; they are not jammed against the walls and “see” a larger opening width to compartment-size ratio,  $D/y_L$ . In contrast, particles jammed against the walls tend to clog the box opening. However, the fraction of the more mobile particles drops fast with increasing  $v_0$ , thus leading to plateaus of effusion rate in the limit  $v_0$ . Figure 1(b) shows that the clogging mechanism works even at low packing fraction, though its impact on effusion is reduced.

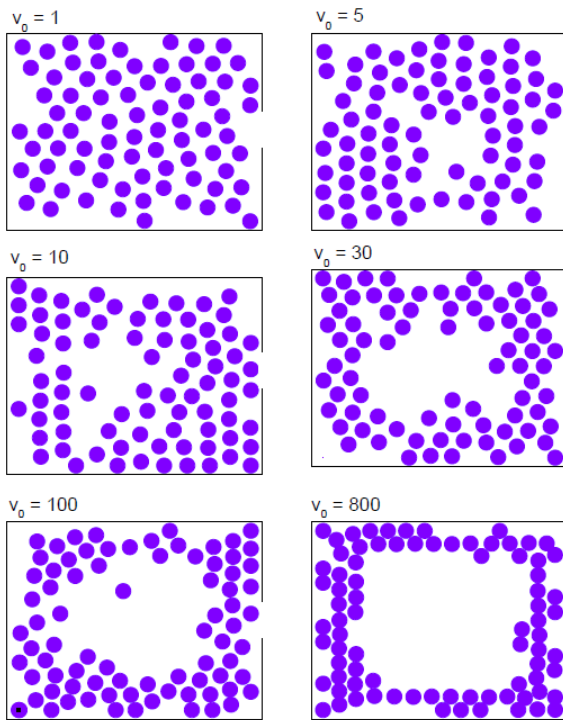
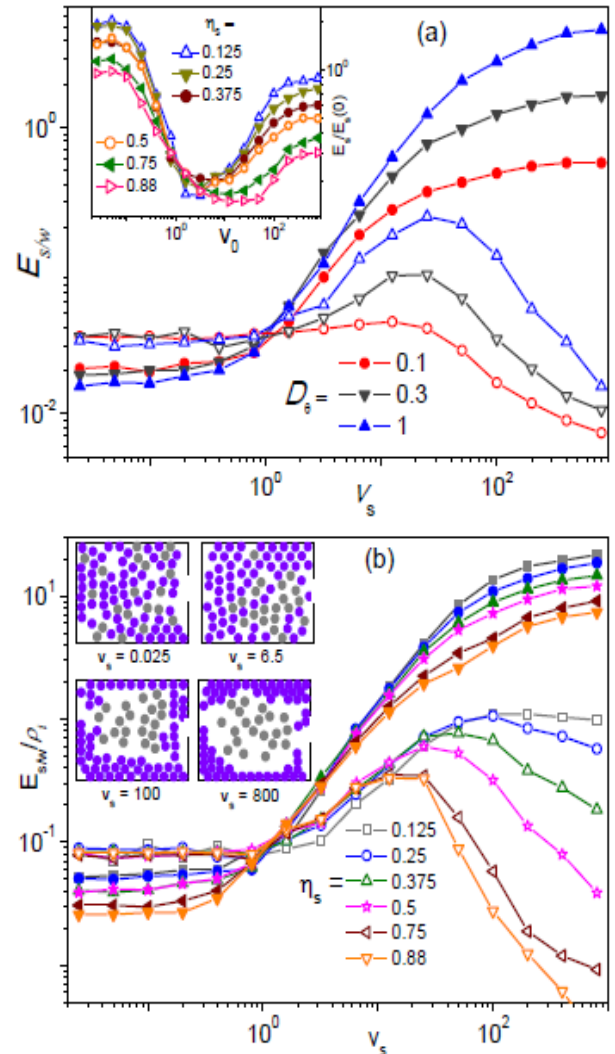


Fig2: Snapshot of phase points of interacting JPs for different self-propelled velocity  $v_0$  and other parameters  $\varepsilon = 1$ , confining length,  $x_L = y_L = 10$ , opening size  $\Delta = 0.5$ , particle radius  $r_0 = 0.5$ ,  $D_0 = 0.03$ , and total particles  $N_t = 80$ .



raising branches of the curves  $E_m(\varepsilon)/E_m(0)$  versus  $v_0$

Fig3: (a) Effusion rates  $E_s$  (filled dots) and  $E_w$  (empty dots) versus  $v_s$  for binary mixture with  $\eta_s = 0.5$  and different values of  $D_0$  (see legends). Inset: Effusion ratio of stronger component,  $E_m(\varepsilon) = E_m(0)$  versus  $v_0$  for different  $\eta_s$  and  $D_0 = 1$ . (b) Effusion rates  $E_s$  (filled dots) and  $E_w$  (empty dots) versus  $v_s$  in a binary mixture for  $D_0 = 1$  and different  $\eta_s$  (see legends). Inset: Snapshot of binary mixture with  $\eta_s = 0.75$ ,  $D_0 = 1$  and different  $v_s$  [0.025 (top-left), 6.5 (top-right), 100 (bottom-left), and 800 (bottom-right)]. Other simulation parameters for main panels and insets:  $v_w = 1$ ;  $\varepsilon = 1$ ,  $x_L = y_L = 10$ ;  $\Delta = 0.5$ ;  $r_0 = 0.5$ ;  $D_0 = 0.03$ ;  $N_m = 80$ .

Figure 3(a) illustrates the dependence of the effusion rates  $E$  of the two active mixture components on their self-propulsion parameters,  $v_0$  and  $D_0$ . The mixture is of 1:1 molar ratio of strongly (s) and weakly (w) active particles. In Fig. 3(b), we examine the consequences of gradually increasing the fraction of guest particle for different values of their self-propulsion speed,  $v_s$ . While all effusion plots exhibit the same general behavior as in Fig 3(a), a few additional features are remarkable:

(i) The effusion rate of the strongly active JPs keeps increasing, but more slowly than  $E_m(0)$  in Fig. 1(a), due to their interaction with the less active JPs. In such a limit, the most active particles tend to push the less active ones against the box walls. Moreover, like in one component systems, clogging effects have great impact on the effusion of both the weak and strong active components.

(ii) On the contrary, the effusion rate of the weak JPs remains unchanged for  $v_s$  up to  $v_w$ ; upon further increasing  $v_s$ , it goes through a maximum in correspondence with the mechanism of effective motility transfer. Again, for very large self-propulsion,  $v_s \gg x_L D_0$ ;  $y_L D_0$ , strongly active JPs jam against the container walls, thus pushing the weaker JPs inside [see snapshots of Fig 3(b) and Fig. 2]. Accordingly, the weaker JPs have a little chance to escape through the opening so that effusion is

drastically suppressed.

Moreover, no decaying branch of  $E_w$  vs.  $v_s$  is detectable at low  $\eta_s$ . This happens because very few strong JPs cannot possibly confine all weak particles in the box' interior.

In conclusion, we stress that adding a small amount of strongly active JPs does suffice to enhance the effusion of sluggish active JPs, but an excess of them can produce the opposite effect! We also remind that, as illustrated by our simulation snapshots, the two components of an active binary mixture can separate into two distinct phases, when the self-propulsion length of one is much larger than the size of the container and the other one much shorter, that is, for  $v_s/D_0 \gg x_L, y_L \gg v_w/D_0$ . However, phase segregation should be avoided for better motility transfer.

As shown in figure 1(b), there is a window of the tunable  $v_s$ , where the effusion rate of the weak particles becomes enhanced by 2 to 7 times, depending on their rotational relaxation time and composition of binary mixture. Also, the span of this window is sensitive to the persistence length of self-propelled motion. This striking result confirms that, even in the absence of inertia, the motility of the more active microswimmers can be effectively transferred to the less active microswimmers. In our numerical analysis we assumed the pore to be centered in one side of a square-shaped simulation box. However, sliding boundary conditions as the JPs move against the cavity walls, can affect their average effusion rate. Our simulation shows that this may become an issue only at zero temperature. As a matter of fact, thermal fluctuations assist the escape mechanism by enhancing particle diffusion along the boundaries, thus suppressing possible effects related to the cavity geometry and the actual pore location. To verify this point, we simulated the effusion rate (not shown) for a modified box geometry, whereby the escape pore was moved toward one corner; for the simulation parameters of Fig. 1 we detected no

appreciable variations of the relevant effusion rates.

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In addition to the research works described above, we are still working on other issues from last year's in particular diffusion control autocatalytic reactions.

## V. Future plan

In the next fiscal year, we plan to explore the following issues:

### (i) Cooperativity in binary mixture of two sort of Janus particles.

Our simulation results in binary mixture suggested that motility transfer in the overdamped regime associated with non-trivial dynamics of weak and strong JPs. We recognized some parameters regime of self-propulsion, where totally different effusion rate through motility transfer is observed. Large variation of effusion associated to change in spatial distribution of phase points, some of which show very crowded and uncooperative motions, while others seem to act as one resulting in better *cooperativity*. Now key issue we want to explore is the degree of cooperativity and its functional dependence on self-propulsion properties of both type of species in binary mixture. In general, cooperativity is quantify by estimating the following quantity.

$$N_X^{Co}(t) = 1 + \frac{\sum_{i \neq j} \langle X_i X_j \rangle}{\sum_i \langle X_i \rangle}$$

Where,  $X_i(t) = r_i(t) - r_i(0)$ , is the displacement of the  $i$ -th particle over time  $t$ . For uncorrelated motion, the above quantifier is one. Positive or negative value associated with some type of correlated motion. To understand in more detail the effects of correlated movement, in addition to the above quantifiers we also want to calculate pair distance and cooperativity of nearest neighbor particles.

### (ii) Escape kinetics of self-propelled Janus particles from circular cavity:

Diffusion is the ubiquitous mechanism for intercellular transport of key bio-molecules that trigger many cellular functions, like cross-membrane ion-current, chemical balance, neuronal signaling etc. The transporting particles diffuse through narrow openings in an impermeable wall and reach at small target. The stochastic escape processes actually play the central role in governing time scale of many cellular functions. The rate of these significant events is largely dictate by geometry of cellular structures in addition to other factors (like, density gradients, temperature, solvation , pH etc) . Therefore, it is desirable to have detailed knowledge about escape rate to gain control over cellular function by tuning external parameters.

In the next, fiscal year we plan to numerically explore narrow escape kinetics for both normal and active Brownian particles. Active Brownian particles, the most fascinating ones are Janus particles, nowadays are designed to use them as a tracer for targeted drug delivery. Thus, the knowledge about their escape processes is of paramount interest. To be specific, here we want to explore escape processes (from circular cavity, see Fig.4) of both normal and active particles through single and/or multiple small openings. We want to understand how the time correlation of the persistent Brownian motion of a JP affects its mean first passage time through one or more cavity openings.



Fig.4: Schematic of some trajectories for Brownian particle in a circular cavity escaping through a very narrow opening.

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To address the above-mentioned issues we will implement and the numerical scheme mentioned in Sec. III.

Currently, I have a “Quick Use” user account and I would like to get extension of computation facilities for next usage term (up to 31<sup>st</sup> March 2021) under the same user category.

## VI. References

- [1] S. Jiang and S. Granick (eds.), *Janus Particle Synthesis, Self-Assembly and Applications* (RSC, Cambridge, 2012).
- [2] C. Bechinger, R. Di Leonardo, H. Löwen, C. Reichhardt, G. Volpe, and G. Volpe, *Active particles in complex and crowded environments*, *Rev. Mod. Phys.*, 2016, 88, 045006
- [3] P. K. Ghosh, V. R. Misko, F. Marchesoni, and F. Nori, *Phys. Rev. Lett.* **110**, 268301 (2013).
- [4] D. Holcman and Z. Schuss, *Stochastic Narrow Escape in Molecular and Cellular Biology* (New York, Springer, 2015).

Usage Report for Fiscal Year 2018

**Fiscal Year 2017 List of Publications Resulting from the Use of the supercomputer**

**[Publication]**

T. Debnath, P. K. Ghosh, Y. Li, F. Marchesoni, and F. Nori, 'Active Diffusion Limited Reactions', The Journal of Chemical Physics , vol-150, article no. -154902, 15 April 2019 ; doi: 10.1063/1.5081125

**[Proceedings, etc.]**

None

**[Oral presentation at an international symposium]**

None

**[Others (Press release, Science lecture for the public)]**

None