Usage Report for Fiscal Year 2019

Project Title:

Lattice calculation for eRHIC physics using Chiral Quark action

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1． Background and purpose of the project

Parton Distribution Function (PDF) is one of the structure functions of hadrons, which links many hadronic scattering cross sections to the short distance, perturbatively computable scattering amplitude between quarks and gluons. In the partonic picture of hadron, PDF can be viewed as the probability density q(x) for finding a particle with a certain longitudinal momentum fraction x. Lattice QCD is the only method that can provide a theoretical determination of the PDFs from the first principle. A lattice calculation of PDF will provide theoretical guidance for EIC experiments, which aims at understanding the structure of hadrons at more comprehensive level. Also, lattice calculation could also provide valuable information for spin dependent PDF, e.g. helicity and transversity, which is experimentally hard to reach.

The purpose of this project is to provide an ab initio determination of the PDFs of the pions and protons using lattice QCD. It is known for quite some time that the moments of PDF can be calculated with local operators on the lattice. However, if one aims at moments of order higher than 3 or 4, the local operator quickly becomes very complicated (e.g. mixes strongly with lower dimensional operator) and lattice calculations with the operator become very noisy. It is then realized that non-local operators shall be used instead. This is a booming field and many new ideas, calculations, and results are emerging. At present, we have lattice calculations by many collaborations including the LP3 collaboration, ETMC collaboration, the Jlab collaboration, and the BNL collaboration. In particular, the iso-vector PDF, which is the difference between the up quark PDF and the down quark PDF, was computed for pion and proton. The spin dependent proton PDFs were also explored.

Different from other lattice PDF calculations. This calculation of project is performed at physical point with Domain wall fermions (DWF). This would allow us to make a quantitative comparison with the phenomenological PDF. Chiral symmetry is persevered by the DWF action. In comparison, all previous computations were using Wilson Clover actions that broke Chiral symmetry. Chiral symmetry will bring us two benefits. (1) Our calculation will be free of discretization error linear in the lattice spacing. The discretization error can only enter at the second order of the lattice spacing. This property significantly reduces the lattice spacing error. For many observables we have studied on this ensemble, the discretization error is only a few percent. (2) For Wilson Clover fermion action, the lattice operator used to measure the quasi-PDF will mix with other unwanted operators. However, this mixing is prohibited by Chiral symmetry, and our proposed calculation will be free of this issue.

Conventionally, calculations using DWF are quite expensive because of the extra dimension introduced. However, the recent developed methods: zMobius + Multi-Grid Lanczos + AMA, reduces the computational cost of DWF by a factor of 100 or 1000, therefore making the calculation with DWF very appealing and competitive.

2． Specific usage status of the system and calculation method

During the FY2019, about 11,000,000 core * hours of the HOKUSAI BigWaterfall (BW-MPC) machine were used. This is the second year of the project and we have accumulated more statistics. The calculation is performed on two lattice ensembles generated by the RBC-UKQCD collaboration. They are the $48^3 \times 96$ Iwasaki ensemble, and the coarser $24^3 \times 64$ cube DSDR ensemble. Both of the ensembles are generated with the physical pion mass, which enable us to make quantitative comparison between the theoretical lattice calculation based on first principle and the phenomenological determination of PDF.

We use the quasi-PDF approach, initially proposed by Xiangdong Ji, in this study. We will need to measure the matrix elements with the following operator

$$\mathcal{O}_{\delta z} = \overline{\psi}(\delta z)\gamma_t U(\delta z, 0)\psi(0)$$

sandwiched between a hadron state, preferably boosted by a large momentum:

$$\tilde{Q}(\delta z, P_z) = \frac{1}{2E_z}\langle P_z|\mathcal{O}_{\delta z}|P_z\rangle.$$

Because of the Wilson line in the operator, the above matrix elements will suffer from linear divergences, which depends on how the Wilson line is implemented on the lattice. In our calculation, we use 1 step HYP smeared gauge link to calculate the Wilson line. The HYP smearing will reduce the linear divergence, but to completely remove it, we calculate the following ratio:

$$\tilde{Q}_R(\delta z, P_z) = \frac{\tilde{Q}(\delta z, P_z)}{\tilde{Q}(\delta z, 0)}.$$

The Fourier transformation of the above function, which is also referred to as the Ioffe-time distribution, is the quasi-PDF:

$$\tilde{q}(x) = \int \frac{P_z d\delta z}{2\pi} e^{-ixP_z\delta z}\tilde{Q}_R(\delta z, P_z).$$

One important property of PDF is its moment, defined by

$$a_{n+1} = \int x^n\, q(x)dx,$$

which can be given by the derivative of the Ioffe-time distribution near the point$\delta z = 0$.

3． Result

As stated above, the calculation is performed on two RBC-UKQCD generated ensembles.

- 48I: lattice size $48^3 \times 96$, inverse lattice spacing 1.73 GeV, physical pion mass 139 MeV. We have calculated 22 configurations. Each with 48 evenly spaced momentum smeared point sources, each source is calculated with 3 different sinks separated from the source by 4, 6, 8 lattice spacings, and each with 5 different momenta, ranging from 0 up to $4 \times 2\pi/L$.
- 24D: lattice size $24^3 \times 64$, inverse lattice spacing 1.015 GeV, physical pion mass 139 MeV. We have calculated 101 configurations. Each with 64 evenly spaced smeared point sources, each source is calculated with 3 different sinks separated from the source by 4, 6, 8 lattice spacings, and each with 5 different momenta, ranging from 0 up to $4 \times 2\pi/L$.

The results from the 24D ensemble is shown below. The curves plotted are the real and imaginary parts of the iso-vector (u-d) unpolarized proton quasi-PDF before Fourier transformation. This is also sometimes referred to as the Ioffe-time distribution.

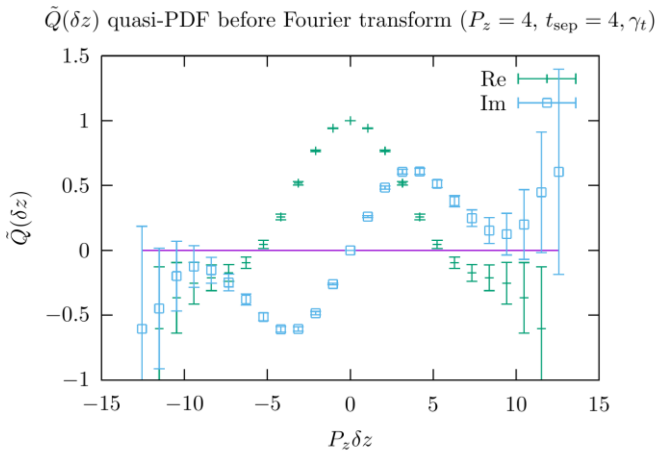

Figure 1: Proton u-d (24D)

The slope of the imaginary part of the above curve at $\delta z = 0$ gives the second moments of PDF, which is the momentum fraction carried by the up quark minus the down quark. This slope can be mostly accurately calculated with source sink separation equal to 6 (about 1.2 fm) to control the excited states effects. With momentum equals 2 lattice momentum ($2 \times 2\pi/L$), we have

$$a_2^{u-d,t_{sep}=1.2fm} = 0.193(18).$$

The uncertainty quoted above is statistical only. This is quite accurate and in agreement with the phenomenological determination. E.g. CT14 gives 0.158(12). We find that proper control over the excited states is essential for obtaining an accurate moment. If we use the data from time separation equal to 4 (about 0.8 fm), we shall instead have

$$a_2^{u-d,t_{sep}=0.8fm} = 0.239(4),$$

which has much smaller statistical error, but much larger excited states contamination.

It should be noted that our value quoted above is simply the slope of the curve $\delta z = 0$. However, to obtain the moments of PDF, one need to perform a matching step to relate the quasi-PDF to PDF. Our preliminary study suggests that the correction is small.

We have also obtained the charged pion valence quasi-PDF. The following plots shows the up quark quasi-PDF before Fourier transformation (only the connected diagram contribution is included, the real part of the curve corresponds to the valence distribution):

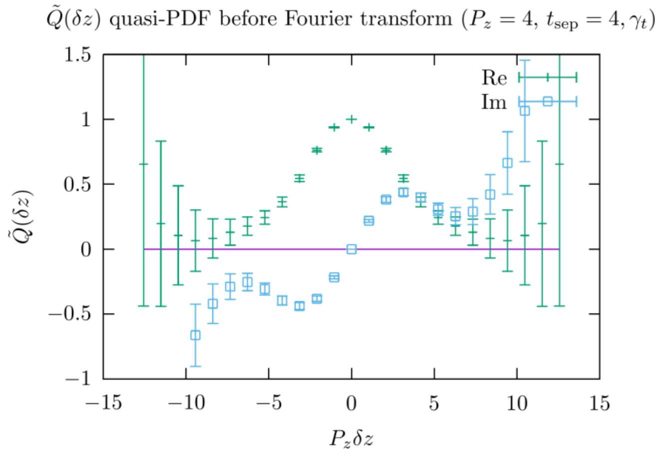

Figure 2: Pion valence (24D)

The results are also obtained with the 48I ensemble. The proton iso-vector quasi-PDF before Fourier transformation is:

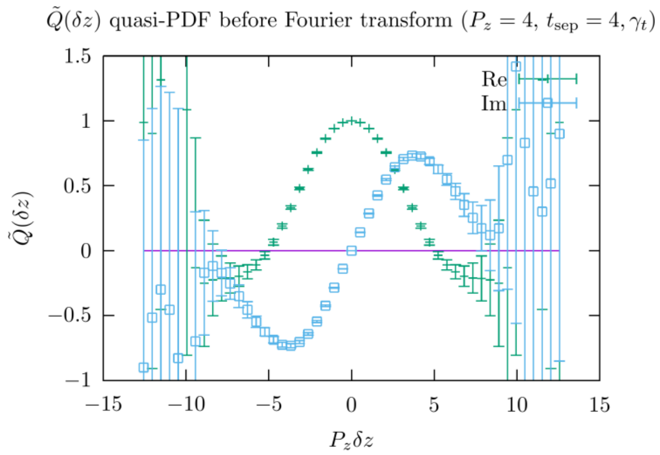

Figure 3: Proton u-d (48I)

The pion up quark quasi-PDF before Fourier transformation is (again, only the connected diagram is included, the real part of the curve corresponds to the valence distribution):

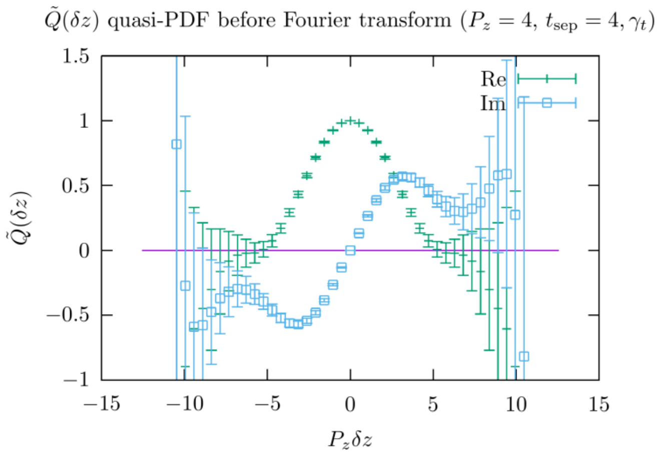

Figure 4: Pion valence (48I)

4. Conclusion

In the project, we have calculated the quasi-PDF for both pion and proton. At present, only the connected diagram is calculated. The Ioffe-time distribution before matching is plotted, which after matching and Fourier transform, give parton distribution function (PDF). In particular, we already obtained preliminary results for the moments of proton iso-vector PDF. The result is found in good agreements with the phenomenological determination. We find that proper control of excited states effects is important in order to obtain a reliable moment, and therefore PDF, from lattice calculation.

5. Schedule and prospect for the future

Comparing with the 24D ensemble calculation, the source sink separation is smaller for 48I. Therefore, the excited states effect is not completely under control even with the largest source sink separation available. However, we expect that a more elaborate analysis with two-state fit can largely fix the problem. This is a direction we are actively working on.

In addition to the spin matrix γ_t in the operator

$$O_{\delta z} = \overline{\psi}(\delta z)\gamma_t U(\delta z, 0)\psi(0),$$

we have also computed the matrix elements for other spin matrices. With these data, we shall perform the corresponding analysis and obtain the two types of polarized PDF: helicity and transversity.

In addition to performing the needed analysis, we also plan to explore improvement both statistical error reduction and development of the quasi-PDF framework. Especially, we would like to explore the possibility to extract more information from quasi-PDF calculated with not so large momentum, or even zero momentum. We also plan to study the closely related subject of pion quasi- distribution amplitude (pion quasi-DA)