

**Project Title:****Precise Determination of Muon  $g-2$  using Chiral Quark action****Name:**

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**Description of the project**

The anomalous magnetic moment,  $g-2$ , of the muon measured at Brookhaven National Laboratory very accurately, 0.54 ppm. The experimental results disagrees from theoretical prediction from the standard model (SM) of particle physics at the level of three standard deviation, which is one of the most compelling hint for new law or new particles of fundamental physics. From 2017, a new  $g-2$  experiment called E989 at Fermi National Accelerator Laboratory, also followed by another  $g-2$  experiment with entirely different methods in Japan, the E34 experiments at J-PARC. The precision target of the new experiments are four time more accurate than that of BNL experiment, would announce its first results in later 2018.

It is urgent and imperative task for theory and computing communities to reduce uncertainties in the theoretical prediction by improving determinations for hadronic contributions.

In this HOKUSAI project, we carried out the first principle lattice QCD calculations with chiral lattice quark, domain-wall fermion, to compute the most dominant source of error,

the hadronic vacuum polarization (HVP) in a new method. This reduces the long-distance noise in our recent  $g-2$  calculation [1] .

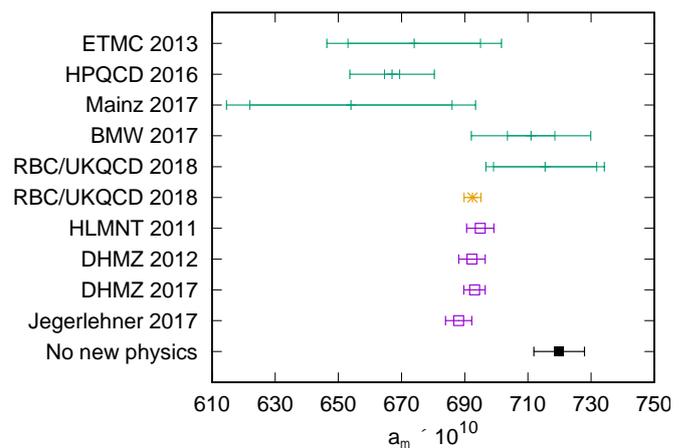


Fig: A compilation of current theoretical efforts for HVP determinations.

In the above figure, Green symbols are from genuine lattice QCD, purple are the dispersive analysis prediction, and the yellow is a combined result of the two. Black is from the BNL experiment in case there is no new physics. Our calculation on HOKUSAI is to improve green symbol by computing correlation functions of various hadron sources, and more accurately determine the long distant part of

vector two points, which corresponds to grey blobs in the figure below.

HVP contribution to the anomalous magnetic moment ( $g-2$ ) is determined by the Green's function of two point-like electro-magnetic currents, which is very noisy in the long distance. By using other operator, whose quantum numbers are same as point-like EM current, one could control, and extract the information of the long-distance of the original EM current. This method is called variational method using the generalized eigen value problem (GEVP) [2].

To make the maximize the benefits of the precious computational resources, we utilized several algorithmic/numerical techniques, including, distillation [3] Domain-compressed eigenvectors [4], All-mode Averaging (AMA)[5], zMobius [6], and efficient code is based on Grid and Grid utilities.

We generated distillation data for a near-physical-point domain-wall fermion ensemble at coarse lattice spacing. We have generated data with 60 eigenvectors of the spacial-laplacian operator measured on all 64 time-slices of the  $24^3 \times 64$  lattice, from which we create various hadron sources. We employ an AMA technique with exact measurements on the first 16 time-slices and sloppy measurements on all 64 time-slices.

We first plot the eigenvalues of energies obtained from a four-operator basis with a point-like local vector current, a smeared vector current, and two  $I=1$  two pion operators with the lowest units of momentum. We contrast

these energies with the Luscher prediction for the  $\pi\pi$   $I=1$  ground state and the infinite-volume  $\rho$  mass parameter. The horizontal axis is the Euclidean time in units of 0.2 fm. We notice good stability for the first three states, out of which the second excited state turns out to be above the  $\rho$  peak energy (770 MeV).

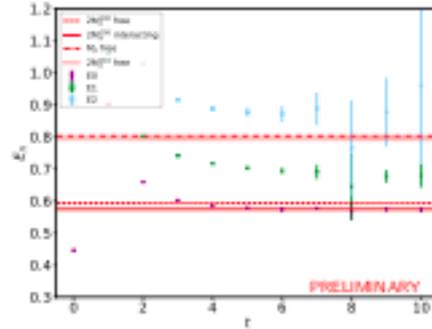


Fig: energy extracted from GEVP using four states (point-like vector current, smeared vector current, and two pion operators) as a function of time separation of Green's functions,  $t$ .

In addition to energy of ground state and excited states, we could also extract the wave functions (or overlap factors) of the states in terms of operators  $O_i$  employed in GEVP,  $\langle 0 | O_i | n \rangle$ . For the muon  $g-2$ , which is defined by the two point function of the point-like local currents, the most important wave function is the ones with the local current,  $c_n = \langle 0 | V_\mu | n \rangle$ , which are plotted below.

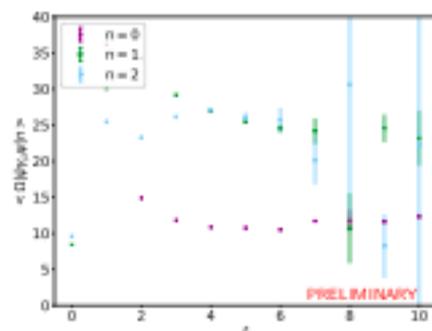


Fig: The wave function of the local current,  $c_n = \langle 0 | V_\mu | n \rangle$  as a function of extracted time separation  $t$ .

The hadronic vacuum polarization contribution to  $g-2$ , from the lowest energy two pion ( $\pi\pi$ ) states are suppressed by  $|c_0|^2 \sim 1/10$ . From energies and wave functions of a few of the lowest states, we could reconstruct the point-like two point function, which significantly help the statistical control at the long distance.

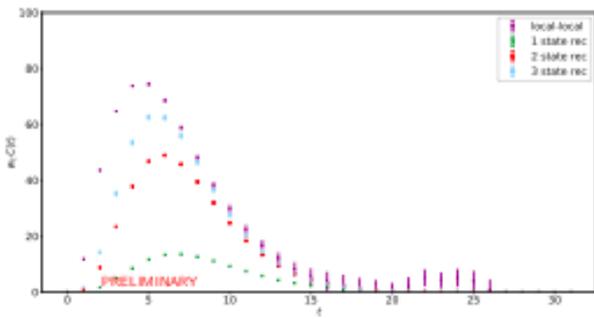


Fig: muon  $g-2$ , HVP contribution integrand as a function of time separation. Purples are for the original point-like current, while other colors are the reconstruction from the GEVP.

This plot shows the total  $g-2$  in purple and the reconstruction from the lowest 1, 2, and 3 states (green, blue, orange). We observe that up to distances of slightly larger than 1 fm the reconstruction is very successful which we can use to reduce the long-distance noise in our  $g-2$  calculations [1]. We expect this improvement to possibly reduce the total uncertainty by a factor of 3. In terms of computational cost, this is about an order of magnitude improvement. This will require to repeat this calculation on additional lattices to control for discretization and finite-volume effects.

## References

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## Usage Report for Fiscal Year 2017

### **Fiscal Year 2017 List of Publications Resulting from the Use of the supercomputer**

#### **[Oral presentation at an international symposium]**

Christoph Lehner, “Results from RBC/UKQCD collaboration”, at Workshop on hadronic vacuum polarization contributions to muon  $g-2$ , Feb 14, 2018.

Aaron Meyer, “LQCD Exclusive Study of  $2\pi$  channel for HVP”, at Workshop on hadronic vacuum polarization contributions to muon  $g-2$ , Feb 14, 2018.