課題名（タイトル）：

光格子における超流動 Fermi 気体のバンド構造

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1. 本課題の研究の背景、目的、関係するプロジェクトとの関係

Ultracold gases in optical lattices provide a new frontier of research where many remarkable phenomena can be observed and investigated. By using Feshbach resonances one can tune the interaction between atoms and investigate the crossover between the Bardeen-Cooper-Schrieffer (BCS) state and a Bose-Einstein condensate (BEC), passing through a resonant regime where the scattering length is very large and the system exhibits universal properties (unitary regime). Here, "universal" means that the properties of the system does not depend on details of interparticle potential. Therefore, study of unitary Fermi gases is also useful for other systems, such as neutron gases in inner crusts of neutron stars, where the neutron-neutron scattering length is much larger than the interparticle separation. Our final aim thus includes learning neutron star crusts by simulating neutron star matter using cold atom gases as well as understanding the cold atom gas systems themselves.

Interplay between the non-linearity and the periodic potential is one of the important issues of the ultracold atomic gases in optical lattices and it gives rise to various intriguing phenomena. However, unlike for BECs, little has been explored so far on this issue for superfluid Fermi gases in optical lattices. One of a handful exceptions is the swallowtail band structure in the BCS-BEC crossover which has been explored by ourselves in our previous project using RICC.

That project on the swallowtail band structure has been finished successfully and our results have been published in the Physical Review Letters [G. Watanabe, S. Yoon, and F. Dalfovo, Phys. Rev. Lett. **107**, 270404 (2011)]. This paper was also selected as a research highlight in Asia Pacific Physics Newsletter.

In 2012, in collaboration with Dr. Nakatsukasa, we started working on period doubling phenomenon in superfluid Fermi gases in optical lattices. Density wave is one of the most fundamental phenomena in condensed matter physics. In Bose-Einstein condensates (BECs) in a periodic potential, non-linearity of the interaction term gives rise to stationary states whose period does not coincide with that of the external potential; instead, a multiple of it. However, it has been found that the period-doubled states in BECs are energetically unfavorable compared to the normal Bloch states and in many cases they are also dynamically unstable.

As for superfluid Fermi gases, on the other hand, there is no study about period doubling and the problem is totally open. Superfluid Fermi gases in the BCS-BEC crossover provide a very intriguing question of how period-doubled states evolve between the BCS and BEC limits. The problem of the period doubling can be even more important in Fermi superfluids, due to the possible implications in superconducting electrons in solids and superfluid neutrons in neutron stars.

In this project, we study states with various periods (mainly, period 1, 2, and 3) of Fermi superfluids in a 1D periodic potential in the BCS-BEC crossover. The first purpose of this project is to explore whether we have period-doubled states in Fermi superfluids. If we find the existence of the period-doubled states in Fermi superfluids, we shall look into how they evolve along the BCS-BEC crossover and solve the following questions.

1) Difference and similarity between Fermi superfluids and BECs. We shall pursue unique properties of period-doubled states of Fermi superfluids which are absent for BECs.

2) Stability of the period-doubled states along the BCS-BEC crossover. The main question is whether we have dynamically stable period-doubled states in some parameter region.

3) Experimental consequences of period-doubled states in the BCS-BEC crossover.

4) A criterion for appearance of the period-doubled states in the BCS side.

2. 具体的な利用内容、計算方法

To study the period-doubling phenomena in ultracold superfluid Fermi gases, we use a mean-field theory based on the Bogoliubov - de Gennes (BdG) equations. We study the whole region along the BCS-BEC crossover including the unitary regime at zero temperature focusing on the situation in which the lattice potential is relatively weak as in the recent experiments. In such a situation, the tight binding description is not adequate and a full numerical approach based on the BdG equations is called for. Although approximate, this approach captures basic features along the whole BCS-BEC crossover, including the formation of molecules and the most challenging unitary limit where, for uniform 3D configurations, the predictions are in reasonably good agreement with ab initio Monte Carlo simulations. The BdG equations apply also to situations where the density varies over distances of the order of the healing length.

To study dynamical stability of period-doubled states, the following two methods can be taken: 1) linear stability analysis based on the framework of quasi-particle random phase approximation (QRPA) and 2) direct simulation of the time evolution by solving the time-dependent BdG equations.

3. 結果

Using numerical simulations based on the BdG equations, we have found that the period-doubled states can be energetically favorable compared to the normal Bloch states in the BCS side. This is in sharp contrast to the situation of the period-doubled states in BECs. Going to the deep BCS regime, the period-doubling nature is mainly possessed by the pairing field, which decreases to zero. We thus speculate that our period-doubled states emerge due to the superfluidity. We have also found that, in some parameter region, the period-doubled states are energetically favorable even compared to period-3 and period-4 states. In addition, we have performed systematic study for a wide range of parameters and examine where the multiple period states appear as energetically stable states.

After we obtained the energetically stable period-doubled states, we started to study their dynamical stability. For this purpose, we have been trying to build a numerical code for QRPA calculations during this year. We are still working on the code building and have not started production run for the stability analysis yet.

4. まとめ

In summary, we continue our project on the period doubling phenomenon in superfluid Fermi gases along the BCS-BEC crossover in collaboration with Dr. Nakatsukasa. We have found that period-doubled states exist as stationary states in Fermi superfluids and it can be energetically favorable compared to normal Bloch states in some parameter region. Dynamical stability analysis of the period-doubled states is highly awaited and we have been struggling to make a numerical code for QRPA calculations. We need to continue this task in the next year.

5. 今後の計画・展望

In the year 2013, we have been working mainly on programming of a QRPA code. So far, unfortunately, we have not manage to make a code which gives correct results. If it takes further much time to make a working QRPA code, we plan to take another method: direct numerical simulations based on the time-dependent BdG equations. Since we already have a working time-dependent BdG solver, the latter might be better way to go. In both the cases, we need large computing resources.

平成 25 年度　RICC 利用研究成果リスト

【論文、学会報告・雑誌などの論文発表】

Gentaro Watanabe and Sukjin Yoon: *Aspects of superfluid cold atomic gases in optical lattices*, JKPS **63**, 839-857 (2013) [**invited review**].

【国際会議などの予稿集、proceeding】

【国際会議、学会などでの口頭発表】

渡辺　元太郎, Sukjin Yoon, Franco Dalfovo, 中務　孝:
光格子における超流動 Fermi 気体の周期倍化現象,
日本物理学会 2013 年 第 68 回年次大会（広島大学、2013 年 3 月 29 日).

Gentaro Watanabe, Sukjin Yoon, Franco Dalfovo, and Takashi Nakatsukasa:
"Period-doubled states of superfluid Fermi gases in optical lattices along the BCS-BEC crossover",
Korean Physical Society 2013 Spring Meeting (Daejeon Convention Center, Daejeon, Korea, 24 April 2013) [**invited**].

Gentaro Watanabe:
"Non-linear phenomena of superfluid Fermi gases in optical lattices",
APCTP-ICTP International Conference on "Nonlinear Dynamics at the Nanoscale" (APCTP, Pohang, Korea, 29　August 2013) [**invited**].

【その他】

渡辺　元太郎:
"APCTP 冷却原子気体グループ",
原子核研究 **58**, 13 (2013).