平成 23 年度 RICC 利用報告書

課題名（タイトル）：

光格子における超流動 Fermi 気体のバンド構造

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1. 本課題の研究の背景、目的、関係するプロジェクトとの関係

Ultracold gases in optical lattices provide a new frontier of research where many remarkable phenomena can be observed and investigated. By using Feshbach resonances one can tune the interaction between atoms and investigate the crossover between the BCS state and a Bose-Einstein condensate (BEC), passing through a resonant regime where the scattering length is very large and the system exhibits universal properties (unitary regime).

Interplay between the non-linearity and the periodic potential is one of the important issues of the ultracold atom gases in optical lattices. For Bose-Einstein condensates (BECs) in optical lattices, it has been pointed out that the non-linearity due to the interaction can change the band structure drastically such that the loop structure called "swallowtail" appears in the energy dispersion of the superfluid flow. Swallowtails have a large impact on transport properties: A direct consequence of the emergence of the swallowtail is a breakdown of Bloch oscillations. Since Bloch oscillations have various important applications, such as precision measurements of forces and controlling the motion of a wave packets, a better understanding the swallowtail is useful in these contexts.

The problem of the swallowtail band structure can be even more important in Fermi superfluids, due to the possible implications in superconducting electrons in solids and superfluid neutrons in neutron stars. However, unlike the Bose case, little has been studied in this problem so far and a fundamental question whether or not the swallowtail exists along the crossover from the Bardeen-Cooper-Schrieffer (BCS) to BEC states is still open.

To solve this problem, we study the band structure of Fermi superfluids in the 1D periodic potential in the BCS-BEC crossover. Main purpose of this project is to explore whether there exist such loop structures in Fermi superfluids. If we find the existence of the swallow-tail band structure in Fermi superfluids, we shall solve the following issues.

1) Difference and similarity between Fermi superfluids and BECs. We shall pursue unique properties of swallow tails of Fermi superfluids which are absent for BECs.

2) Experimental consequences of swallow tails in the BCS-BEC crossover.

3) A criterion for appearance of the swallow tails in the BCS side.

2. 具体的な利用内容、計算方法

In this project, in order to investigate the band structure of ultracold superfluid Fermi gases, we use a mean-field theory based on the Bogoliubov - de Gennes (BdG) equations. We study the whole region along the BCS-BEC crossover including the unitary regime at zero temperature focusing on the situation in which the lattice potential is relatively weak as in the recent experiments. In such a situation, the tight binding description is not adequate and a full numerical approach based on the BdG equations is called for. Although approximate, this approach captures basic features along the whole BCS-BEC crossover, including the formation of molecules and the most challenging unitary limit where, for uniform 3D configurations, the predictions are in reasonably good agreement with ab initio Monte Carlo simulations. The BdG equations apply also to situations where the density varies over distances of the order of the healing length.

3. 結果

Using numerical simulations based on the BdG equations, we have shown that, in each side of the crossover region, the swallowtail appears in the Bloch energy band of the superfluid above a critical value of the interaction strength. The width of the swallowtail is largest near unitarity, and across the critical value of interaction strength, the profiles of density and pairing field change more drastically in the BCS side than in the BEC side.

More interestingly, we have found that, along with the appearance of the swallowtail in the BCS side, there exists a narrow band in the quasiparticle energy spectrum close to the chemical potential and the incompressibility of the Fermi gas experiences a profound dip, unlike in the BEC side. The emergence of the swallowtail and the dip of the incompressibility can be explained as a consequence of the fact that the chemical potential touches the top of the narrow band in the quasiparticle spectrum when the swallowtail is on the edge of appearing.

4. まとめ

In summary, we have established the existence of swallowtails in the energy band of superfluid fermions in a lattice and have pointed out some key features which make these swallowtails different from those in a BEC. The results are obtained within a range of parameters compatible with current experiments. Our research has been recently published in the Physical Review Letters [G. Watanabe, S. Yoon, and F. Dalfovo, Phys. Rev. Lett. **107**, 270404 (2011)].

5. 今後の計画・展望

It would be interesting to study the possibility of creating a directed motion using the swallowtail band structure. By sweeping the magnetic field across the critical value for the appearance of the swallowtail, one can change the shape of the lowest energy band structure between a sinusoidal form and a quadratic-like form. If one periodically sweeps the magnetic field and synchronizes the period of this modulation with the period of the Bloch oscillation, one could realize a directed motion. Whether this scheme works when one takes account of the adiabaticity is an important question to be clarified.

6. RICC の継続利用を希望の場合は、これまで利用した状況（どの程度研究が進んだか、研究においてどこまで計算出来て、何が出来ていないか）や、継続して利用する際に行う具体的な内容

As we mentioned in the last section, we are planing to examine the possibility of the directed motion. Periodic doubling in the BCS-BEC crossover due to the non-linearity caused by the pairing field is another interesting problem. These future works will be in collaboration with Theoretical Nuclear Physics Group (group leader: Dr. Nakatsukasa).

平成 23 年度　RICC 利用研究成果リスト

【論文、学会報告・雑誌などの論文発表】

Gentaro Watanabe, Sukjin Yoon, and Franco Dalfovo: *Swallowtail Band Structure of the Superfluid Fermi Gas in an Optical Lattice*, Phys. Rev. Lett. **107**, 270404 (2011).

【国際会議などの予稿集、proceeding】

【国際会議、学会などでの口頭発表】

Gentaro Watanabe, Sukjin Yoon, and Franco Dalfovo: *Swallowtail band structure of the superfluid Fermi gas in an optical lattice*, Korean Physical Society Autumn Meeting 2011 (BEXCO, Busan, Korea, 20 October 2011).

Gentaro Watanabe: *Swallowtail band structure in the BCS-BEC crossover*, 2012 First QS2C Theory Forum on "Recent trends in condensed matter physics", (RIKEN, Japan, 15 Jan. 2012) [invited].

【その他】

Gentaro Watanabe, Sukjin Yoon, and Franco Dalfovo: *Swallowtail band structure of the superfluid Fermi gas in an optical lattice*, 16th International Conference on Recent Progress in Many-Body Theories (Barilochee, Argentina, 28 Nov. to 2 Dec. 2011; poster presentation).