SIMULATION OF UNSTEADY SEPARATED FLOWS AND FLOW-ADAPTIVE GRID GENERATION

Urmila Ghia
Computational Fluid Dynamics Research Laboratory
Department of Mechanical Engineering
University of Cincinnati, Cincinnati, Ohio 45221-0072

Presented at

7th International Nobeyama Workshop on CFD
Tokyo, Japan
September -24, 2009

Outline

- Remarks in honor of Professor Kuwahara
- Adaptive grids for
  - steady flow
  - time-evolving flow
- Closure

UNIVERSITY OF Cincinnati

Acknowledgements

- Colleagues
- Students
- Funding Agencies
- Ohio Supercomputer Center

UNIVERSITY OF Cincinnati

UC CFDRL

Tribute to Professor Kuwahara

- Ohayoo gozaimasu. Good morning!
- We feel deeply privileged to have this opportunity to express our respects and admiration for Professor Kuwahara.
- He was an accomplished fluid dynamicist with enormous creativity in devising computational approaches suitable for solving very large-scale flow problems of practical interest, as well as fundamental flow problems that revealed basic fluid dynamics phenomena.   Examples:
  - Drag crisis for flow past cylinder
  - Flow in tornadoes
  - Fluid-structure interaction problems
- He devised the diagonal alternating-direction implicit (ADI) scheme that accelerated convergence rate of the numerical solution significantly.

Tribute to Professor Kuwahara (cont'd)

- Professor Kuwahara's commitment to his work was enormous. He carried his personal computer with him, and it continued to do the calculations even as Professor Kuwahara walked around at conferences. Professor Kuwahara had the outer case of his PC modified, so it had a handle to facilitate carrying the PC around.
- He personally owned three supercomputers at his ICFD (Institute for Computational Fluid Dynamics) which flourished with scientific activities with its several Ph.D.- and MS-level employees.
- He was also an accomplished classical pianist, and we often enjoyed his piano playing at his home.

Tribute to Professor Kuwahara (cont'd)

- He was a gracious host. At the Nobeyama conferences he organized, he paid great addition to every detail, not just for the conference attendees, but also for their accompanying families.
- In the 1980s and early 1990s, we always traveled to the Nobeyama conferences with our children Tina and Kiran. The timings sometimes coincided with our daughter Kiran's birthday, September 4.
  - One year at the Nobeyama conference Banquet, the pianist was performing, and then suddenly we heard the happy birthday song, and Mrs. Kuwahara swiftly walked over to Kiran, presented her a bouquet of flowers and a gift box. The gift was a very cute picture frame. We have placed a family photo in it, and the frame sits in our curio cabinet in our family whom.
  - It is a constant reminder of the warm generosity of Professor Kuwahara, and of Mrs. Kuwahara's kind thoughtfulness.

Tribute to Professor Kuwahara (cont'd)

- In 2007, Kiran and her husband Rahul visited Japan, and spent some time with Professor Kuwahara and his family.
- They too commented on Professor Kuwahara's warm-heartedness and generosity. They had dinner with the some of Professor Kuwahara's family the first day, and a lunch with Professor Kuwahara the next day.
- When we saw Professor Kuwahara next at a conference in the US, he too observed how nice a young lady Kiran had grown up to be, and how he liked Rahul a lot.

- Thank you, Mrs. Nagako-san, and Moroe-son, Matate-son and Tabito-son, for sharing Professor Kunio Kuwahara with us for the times we were together. It was an honor.

Professor Kuwahara and Family with Kiran and Rahul (Ghia Family)

UNIVERSITY OF Cincinnati

Tina and Todd with son Jackson (Ghia Family)

Example of Effect of Grid on Solution

Nonlinear Burgers' Equation in x-domain: $$u_{xx} + P(x,u)u_x + R(x,u)u = S(x) + Tu_t$$

where $P = -(u - U)\mathrm{Re},\ R = 0,\ S = 0,\ T = \mathrm{Re}$.

Boundary Conditions: $$u(-\infty,t) = 1,\ u(\infty,t) = 0 \ .$$

Initial Conditions:
$$u(x,0) = \begin{array}{lll} 1.0 & \text{for} & -5 \le x < 0 \\ 0.5 & \text{for} & x = 0 \\ 0.0 & \text{for} & 0 < x \le 5 \end{array}$$

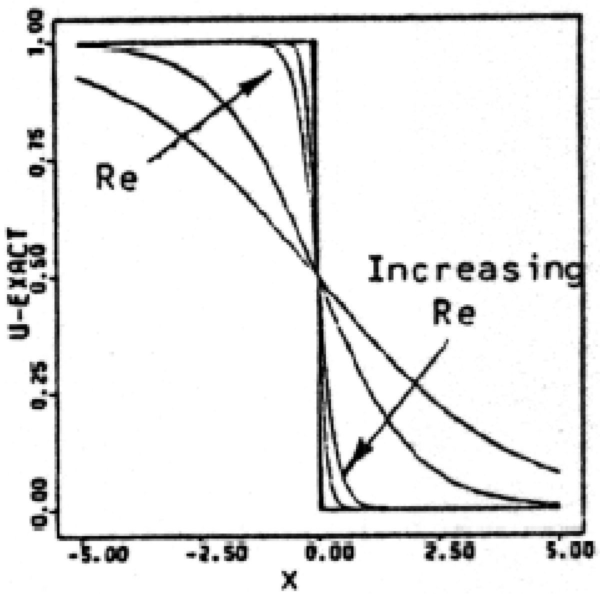
Exact Solution: $$u(x) = U\left[1 - \tanh\left(\frac{U\,\mathrm{Re}\,x}{2}\right)\right]$$

UNIVERSITY OF Cincinnati

UC CFDRL

Example of Effect of Grid on Solution (Cont'd)

Analytical Solution of 1-D Burgers' Equation for increasing Reynolds number



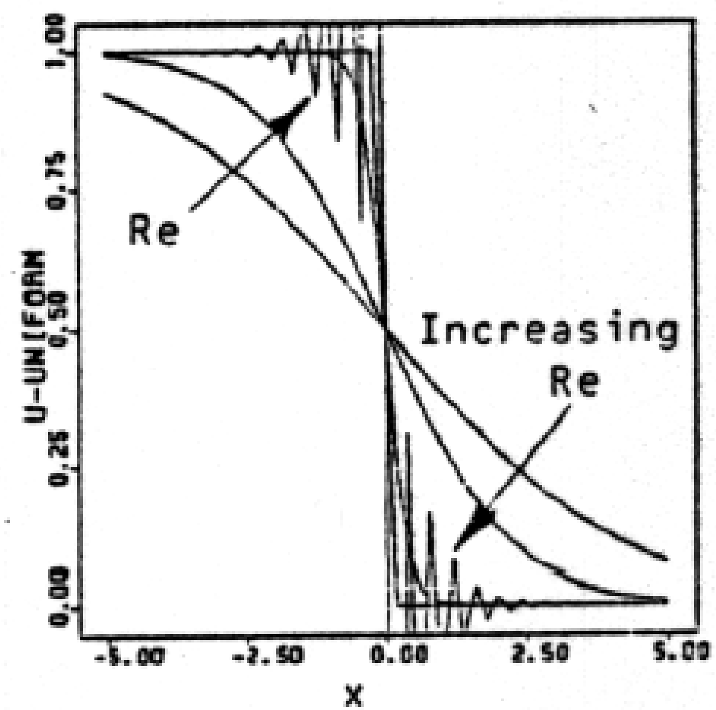
Re	=	1
		2
		10
		100
		200
		1000
		2000
		10000

U. Ghia, K. Ghia, Shin (1983)

Example of Effect of Grid on Solution (Cont'd)

Numerical Solution of 1-D Burgers' Equation for increasing Reynolds number –

51 uniformly spaced grid points



Re = 1
2
10
100
200
1000
2000
10000

U. Ghia, K. Ghia, Shin (1983)

Introduction – Adaptive Grids

- CFD provides numerical solution at discrete locations.
- How to choose these locations is the major question.

- MSU ERC
- Textbooks

Introduction (Cont’d)

- Requirements to satisfy:
  - Boundary-aligned to geometry
  - Spacing must vary smoothly
  - Spacing must resolve critical regions
    - At boundaries
    - In the interior
  - Capture local solutions

Introduction (Cont'd)

- Depending on the application, any single one of these requirements can be a challenge in itself, and in that case, it alone then becomes the only focus of the effort.
- When the selected requirement is simple, we proceed to incorporate one or more of the other requirements into the effort.

Definition of Adaptive Grid

- In a sense, grids that satisfy one or more of the listed requirements are adaptive grids.
- However, the term is usually used in the context of adaptation to flow features.

Introduction (Cont'd): Smooth Spacing Variation

For $y = y(\eta)$:

$$\frac{d^2u}{dy^2} = \frac{d}{dy}\left[\frac{du}{d\eta}\frac{d\eta}{dy}\right]$$

$$= \frac{d}{d\eta}\left[\frac{du}{d\eta}\right]\left(\frac{d\eta}{dy}\right)^2 + \frac{du}{d\eta}\frac{d^2\eta}{dy^2}$$

$$= \frac{d^2u}{d\eta^2}\left(\frac{d\eta}{dy}\right)^2 + \left[\left(\frac{du}{d\eta}\right)_{\text{discretized}} + TE\right]\frac{d^2\eta}{dy^2}$$

$$= \frac{d^2u}{d\eta^2}\left(\frac{d\eta}{dy}\right)^2 + \left(\frac{du}{d\eta}\right)_{\text{discretized}}\frac{d^2\eta}{dy^2} + [TE]\frac{d^2\eta}{dy^2}$$

If the grid-point spacing variation is not smooth, $\frac{d^2\eta}{dy^2}$

is large, and can increase the total truncation error significantly.

UNIVERSITY OF Cincinnati

UC CFDRL

One-Dimensional Nonlinear Burgers' Equation

Burgers' Equation in x-domain:

$$u_{xx} + P(x,u)\,u_x + R(x,u)\,u = S(x) + Tu_t$$

where

$$P = -(u - U)\mathrm{Re},\ \ R = 0,\ \ S = 0,\ \ T = \mathrm{Re}\,.$$

Burgers' Equation in ξ-domain:

$$u_{\xi\xi}\xi_x^2 + \left[\xi_{xx} + P\xi_x - T\xi_t\right]u_\xi + Ru = S + Tu_\tau$$

$$u_{\xi\xi} + \left[x_{\xi\xi} - (P + Tx_\tau)\,x_\xi^2\right]\frac{u_\xi}{x_\xi} + J^2 Ru = J^2\left(S + Tu_\tau\right)$$

$$\mathbf{J} = \mathbf{x}_\xi = \frac{\mathbf{1}}{\xi_{\mathbf{x}}}$$

UNIVERSITY OF Cincinnati

CFDRL

One-Dimensional Nonlinear Burgers' Equation (Cont'd)

Grid Equation: $$x_{\xi\xi} - \left(P + T x_\tau\right) x_\xi^2 = 0$$

Reduced Transformed Burgers' Equation: $$u_{\xi\xi} - R_c \frac{1}{x_\xi} u_\xi + J^2 R u = J^2 \left(S + T u_\tau\right)$$

where

$$R_c = \text{Residue in } \left[x_{\xi\xi} - \left(P + T x_\tau\right) x_\xi^2\right]$$

u_ξ is central differenced,

x_ξ is upwind differenced in the grid equation, and central differenced in R_c.

UNIVERSITY OF Cincinnati

UC CFDRL

One-Dimensional Nonlinear Burgers' Equation (Cont'd) – Solution Algorithm

1. The first-order derivative in the grid equation is approximated using upwind differencing for the term x_ξ .
2. Using this instantaneous grid solution, the residue R_c is computed using central differencing for the term x_ξ .
3. At this time instant, the flow equation is solved using central differencing for all spatial derivatives.

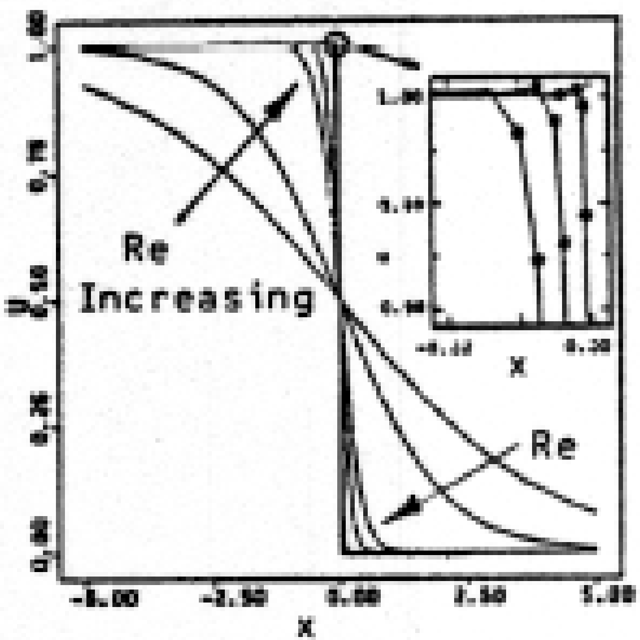
This sequence is repeated, to obtain both the adaptive grid and the flow solution on this grid.

UNIVERSITY OF Cincinnati

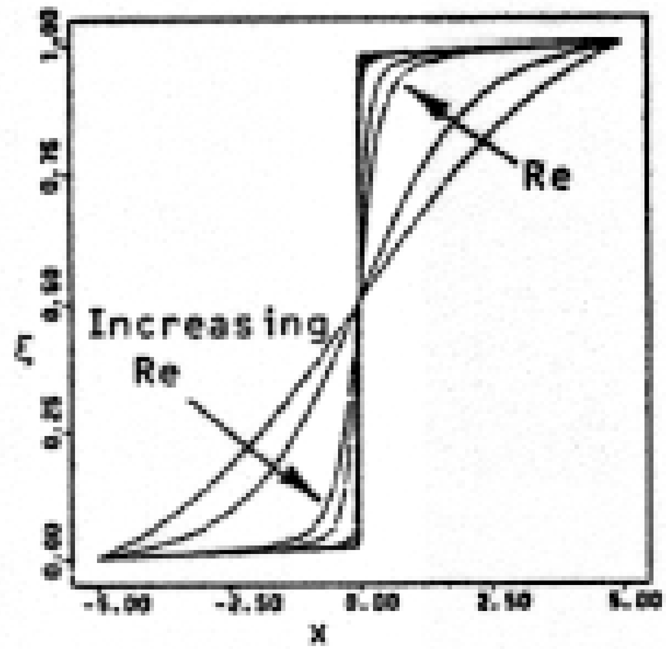
UC CFDRL

1-D Burgers' Equation –
Numerical Solution using Flow-Adaptive Grids

Adaptive Grid and Flow Solution



Computed Velocity Profile

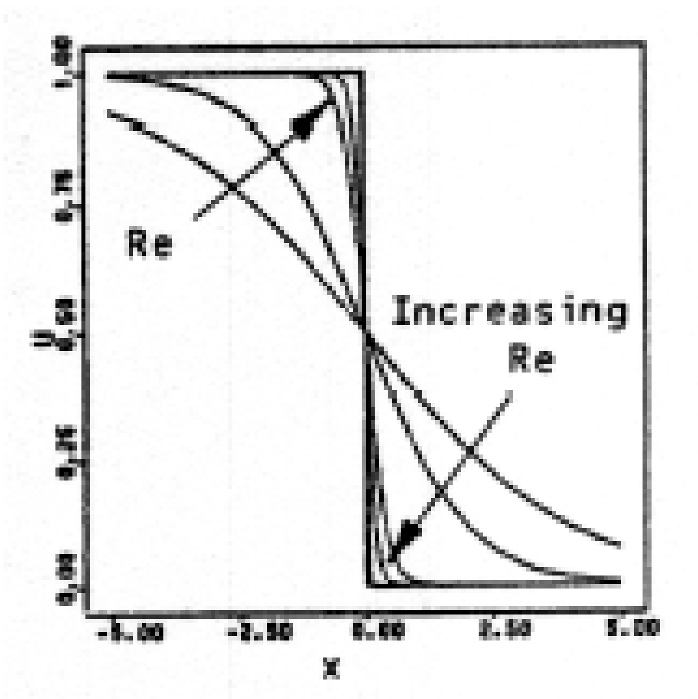


Coordinate Transformation

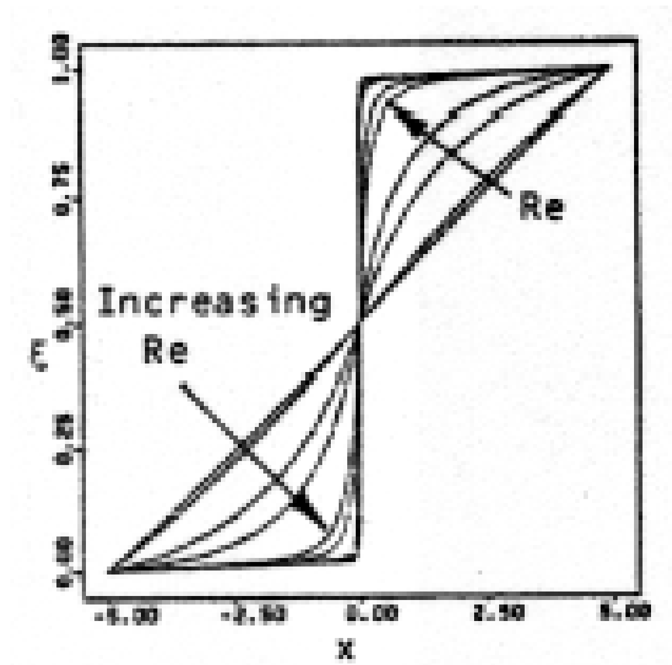
Re = 1
2
10
100
200
1000
2000
10000

1-D Burgers' Equation –
Numerical Solution using Flow-Adaptive Grids

Adaptive Grid (Modified) and Flow Solution



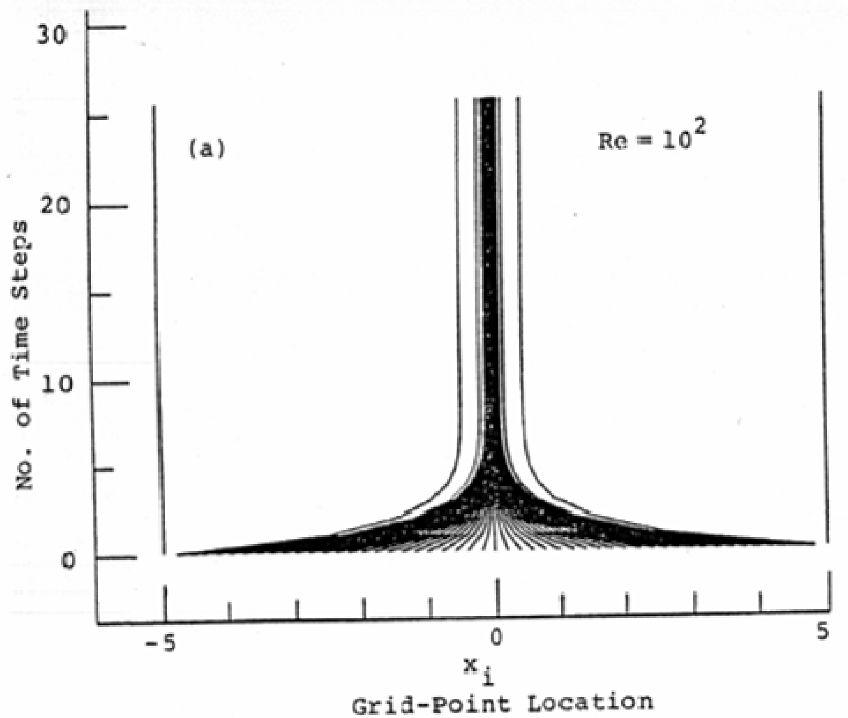
Computed Velocity Profile



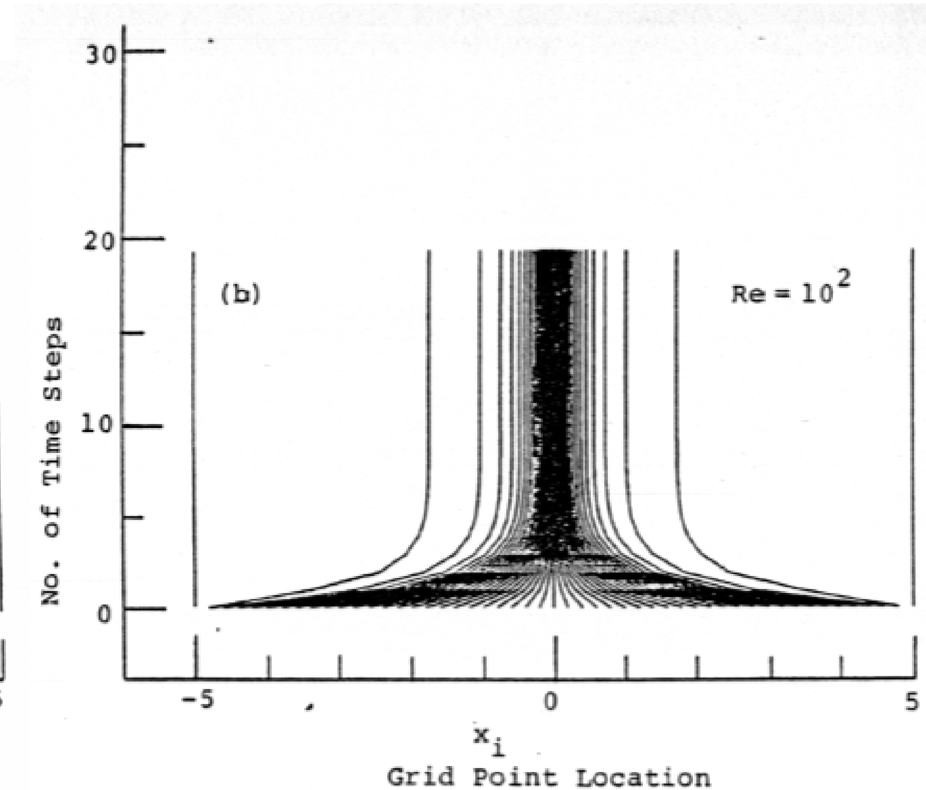
Coordinate Transformation

Re = 1
2
10
100
200
1000
2000
10000

1-D Burgers' Equation – Grid-Point Paths



Adaptive Grid



Adaptive Grid (Modified)

Time-Dependent Flow-Adaptive Grids

Elliptic Grid Generation - Background

$$\phi + i\psi = f(x + iy)$$

$$\phi_{xx} + \phi_{yy} = 0$$

$$\psi_{xx} + \psi_{yy} = 0$$

$$\phi_{xx} + \phi_{yy} = \tilde{P}$$

$$\psi_{xx} + \psi_{yy} = \tilde{Q}$$

UNIVERSITY OF Cincinnati

CFDRL

Elliptic Grid Generation (Cont'd)

Inverted Equations:

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = -\left[P x_{\xi} + Q x_{\eta}\right]$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = -\left[P y_{\xi} + Q y_{\eta}\right]$$

where

$$\alpha = x_{\eta}^2 + y_{\eta}^2$$

$$\beta = x_{\xi} x_{\eta} + y_{\xi} y_{\eta}$$

$$\gamma = x_{\xi}^2 + y_{\xi}^2$$

$$P = J^2 \tilde{P}, \quad Q = J^2 \tilde{Q}$$

$$J = x_{\xi} y_{\eta} - x_{\eta} y_{\xi}$$

Inclusion of x_{τ} and y_{τ} on RHS makes these equations parabolic in time.

UNIVERSITY OF Cincinnati

CFDRL

Adaptive Grids

Truncation error is of the form: $\Delta x^{n} f^{(n+1)}$

Goal is to achieve $(\Delta x) w = \text{constant}$, i.e., $x_{\xi} \mathbf{w} = \text{constant}.$

Differentiation with respect to ξ leads to

$$x_{\xi\xi} w + x_{\xi} w_{\xi} = 0 \quad \textbf{i.e.,} \quad \frac{x_{\xi\xi}}{x_{\xi}} = -\frac{\mathbf{w}_{\xi}}{\mathbf{w}}$$

Recall: $x_{\xi\xi} + \mathbf{P}\mathbf{x}_{\xi} = \mathbf{0}$ **or** $\dfrac{x_{\xi\xi}}{x_{\xi}} = -\mathbf{P}$.

Comparison suggests that P is related to the weight function as

$$P = \frac{W_{\xi}}{W}.$$

Applying this concept to each coordinate direction, $\mathbf{P}_{\xi^k} = \dfrac{w^k_{\xi^k}}{w^k}$

Adaptive Grids (Cont'd)

$$w^k_{i,j} = 1.0 + a^k \frac{\left|\omega_{i,j}\right|}{\left|\omega_{i,j}\right|_{max}} + b^k \frac{\left|\left(\omega_{\xi^k}\right)_{i,j} \big/ \left(\omega_{i,j} + \varepsilon\right)\right|}{\left|\left(\omega_{\xi^k}\right)_{i,j} \big/ \left(\omega_{i,j} + \varepsilon\right)\right|_{max}}$$

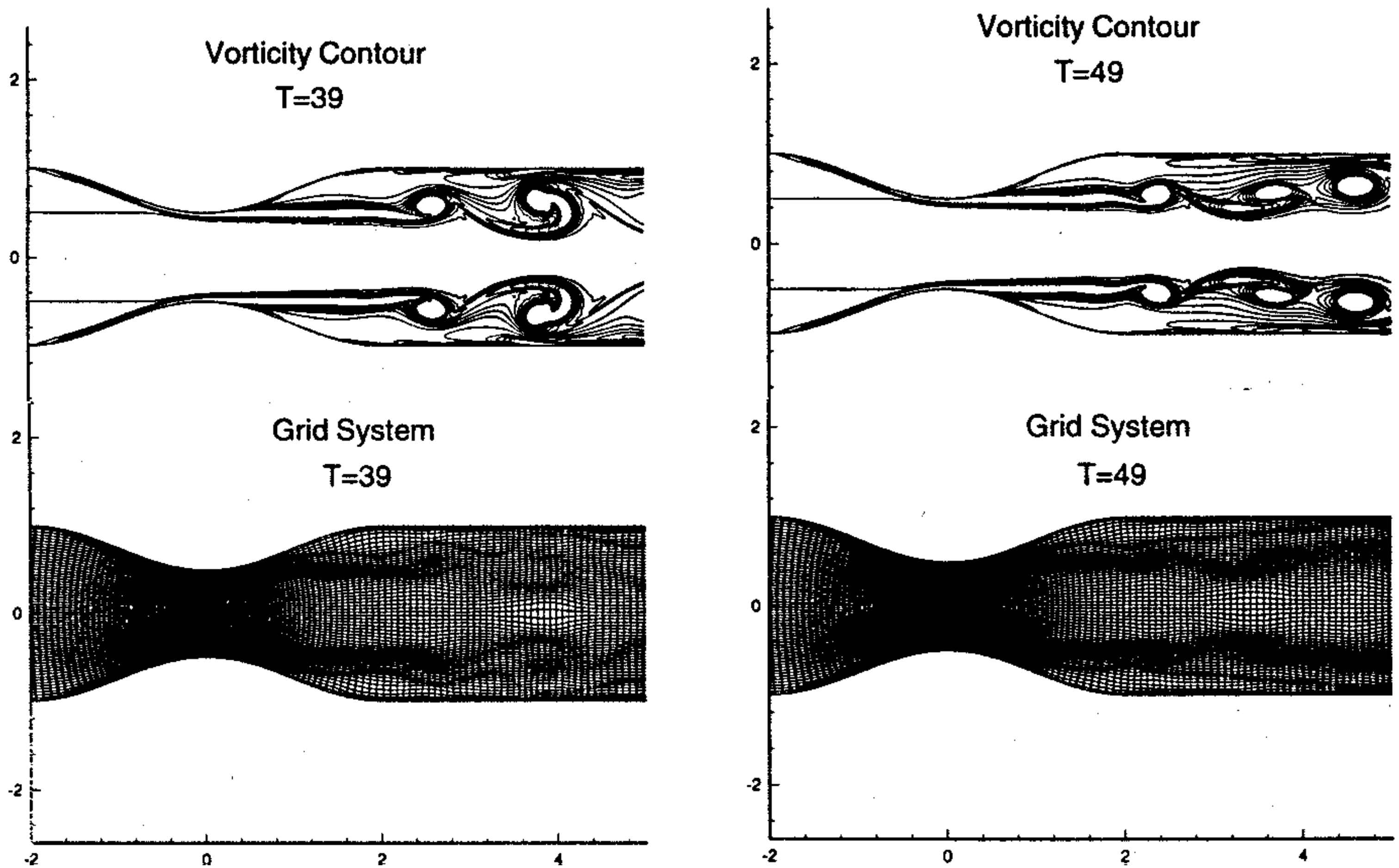
$$+ c^k \frac{\left|\left(\omega_{\xi^k \xi^k}\right)_{i,j} / \left(\omega_{i,j} + \varepsilon\right)\right|}{\left|\left(\omega_{\xi^k \xi^k}\right)_{i,j} / \left(\omega_{i,j} + \varepsilon\right)\right|_{max}}$$

$$\frac{\left(w^i\right)_{min}}{\left(w^i\right)_{max}} = \frac{\left(\Delta x^i\right)_{max}}{\left(\Delta x^i\right)_{min}} \approx 200$$

UNIVERSITY OF Cincinnati

UC CFDRL

Time-Dependent Flow in Stenosis



Closure

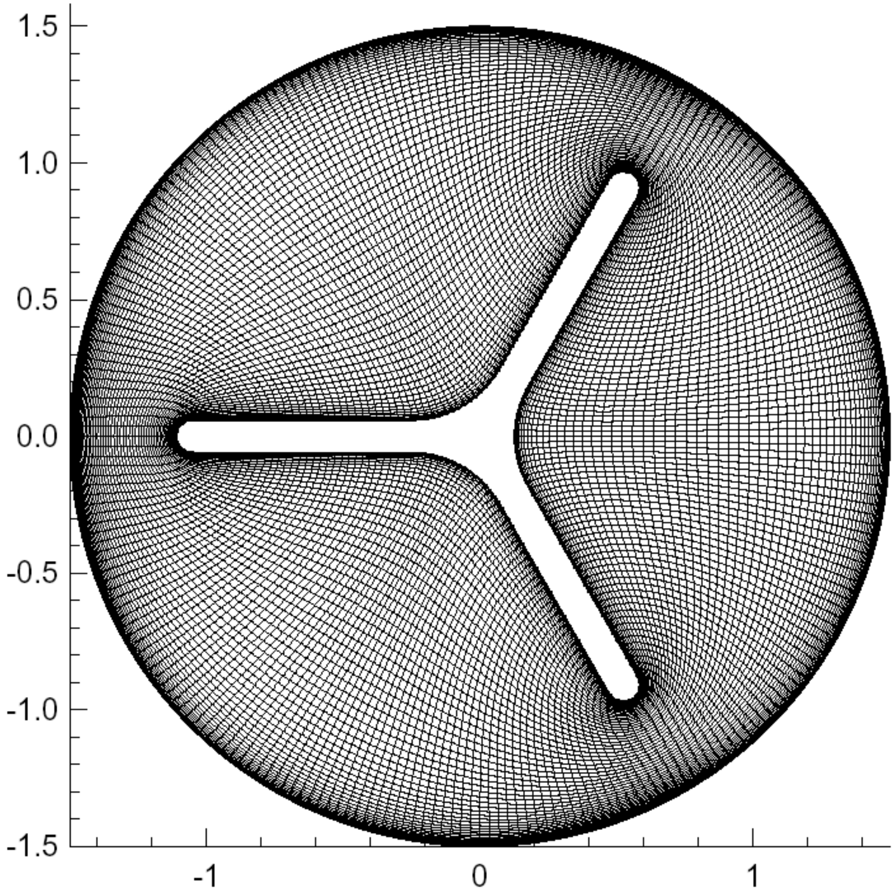
- The incorporation of physics and analytics has shown to produce accurate solutions efficiently.
- Adaption to flow features greatly enhances solution convergence.
- Professor Kuwahara had great influence in accurate computation of fluid flows.

The science of grid generation continues to be an art, and requires blending theoretical rigor with experience and compromise, depending upon problem complexity.

Thank you !

Animation

Grid for 3-fin Mixer



Grid: 265 x 65

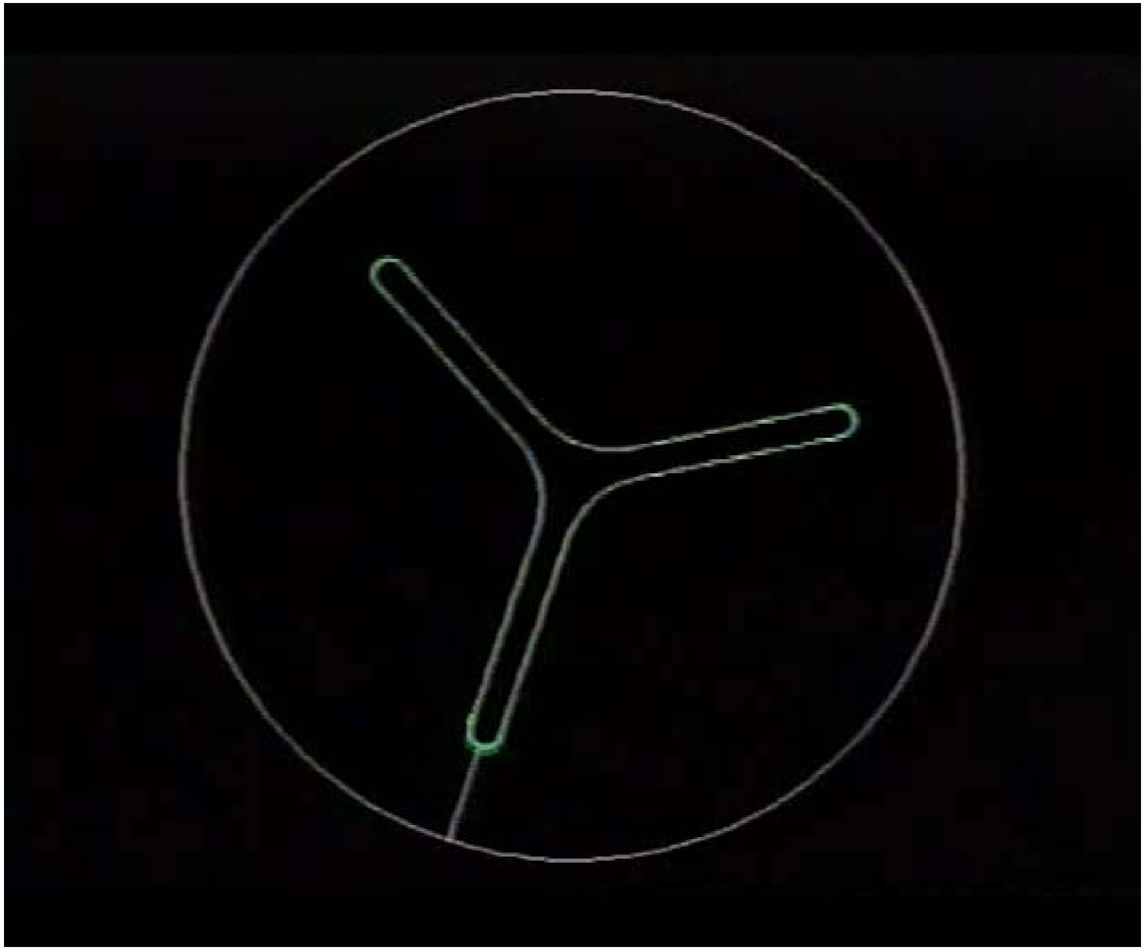
Noll (1999)

UNIVERSITY OF Cincinnati

CFDRL

Example of an Adaptive-Grid Flow Solution – Three-Fin Mixer

Re = 19,387, M = 0.44, Grid: 265 x 65, $\pm 72^{0}$ oscillation



Noll (1999)