IDR-based SOR(s) Method

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An Outline of My Talk

- Introduction (Background and objectives)
- IDR(Induced Dimension Reduction) methods
  - IDR(s) method by Sonneveld and van Gijzen (2008)
  - IDR(s) family methods with parameter L (2008, 2009)
  - I-Jacobi, IGS(Gauss-Seidel) and ISOR methods
  - ISOR(s) method
- Numerical Experiments
- Concluding Remarks

Background and Objectives

- The classical SOR (Successive Over-Relaxation) method is originated from the dissertation by D. Young in 1950. After that, the SOR method has been often used for the solution of problems which stem from various applications.
- The SOR method, however, has many issues on possibility of the solution. Since the SOR method greatly depends on spectrum of iteration matrix, applicability of the SOR method is not robust.
- In my talk, we extend IDR (Induced Dimension Reduction) Theorem proposed by Sonneveld and van Gijzen to designing of the residual of the SOR method, and accelerate its convergence rate and stability.
- Through numerical experiments, we make reveal significant effect of accelerated residual of IDR-based Jacobi, Gauss-Seidel, SOR and SOR(s) methods with parameter s.
- We refer to them as I-Jacobi, IGS, ISOR and ISOR(s) methods, respectively.

References

[1] Sonneveld, P., AGS - IDR - CGS - BiCGSTAB - IDR(s): The circle closed, A case of serendipity, The Proc. of Int. Kyoto Forum 2008 on Krylov subspace methods, pp.1-14, September, 2008.

[2] Sonneveld, P., van Gijzen, M.B., IDR(s): a family of simple and fast algorithms for solving large nonsymmetric linear systems, SIAM J. Sci. Comput., Vol. 31, No.2, pp.1035-1062, 2008.

[3] Wesseling, P., Sonneveld, P., Numerical Experiments with a Multiple Grid- and a Preconditioned Lanczos Type Methods, Lecture Notes in Math., Springer, No.771, pp.543-562, 1980.

[4] Fujino, S., Sonneveld, P. and van Gijzen, M., ISOR(s) method, SIAM Monterey, 26th-29th, Oct., 2009.

A family of IDR(s) methods

- MR_IDR(s) method (van Gijzen, Sonneveld, 2008)
- Bi_IDR(s) method (van Gijzen, Sonneveld, 2008)

- GIDR(s,L) method (Tanio, Sugihara, 2008)
- IDRStab(s,L) method (Sleijpen, van Gijzen and Sonneveld, 2009)
- BiCGStab(s,L), GBiCGStab(s,L) methods (Tanio, Sugihara, 2009)

Algorithm of IDR-based methods

Let $\boldsymbol{x}_0$ be an initial solution,

put $\boldsymbol{r}_0 = \boldsymbol{b} - \mathrm{A}\boldsymbol{x}_0$,

set $\gamma_0 = 0$,

for $k = 0, 1, 2, \ldots$

$$\boldsymbol{s}_k = M^{-1}(\boldsymbol{r}_k + \gamma_k d\boldsymbol{r}_k),$$

$$d\boldsymbol{x}_{k+1} = \boldsymbol{s}_k + \gamma_k d\boldsymbol{x}_k,$$

$$d\boldsymbol{r}_{k+1} = N\boldsymbol{s}_k - \boldsymbol{r}_k,$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{r}_k + d\boldsymbol{r}_{k+1},$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + d\boldsymbol{x}_{k+1},$$

if $||\boldsymbol{r}_{k+1}||_2/||\boldsymbol{r}_0||_2 \leq \epsilon$ stop

$$\gamma_{k+1} = \frac{(\boldsymbol{p}, \boldsymbol{r}_{k+1})}{(\boldsymbol{p}, d\boldsymbol{r}_{k+1})},$$

end for.

Computational cost per one iteration

method	$A\boldsymbol{v}$ $(\times 2nnz)$	$(\boldsymbol{u},\boldsymbol{v})$ $(\times 2n)$	$\boldsymbol{u}\pm\boldsymbol{v}$ $(\times n)$	$\alpha\boldsymbol{v}$ $(\times n)$
SOR	1*	0	3*	2
IGS	1	2	5	2
ISOR	1	2	5	2
BiCGStab	2	4	6	6
GPBiCG	2	7	16	13
BiCGSafe	2	7	14	13

Numerical experiments

- All computations were done in double precision floating point arithmetics.
- Computations were performed on HP xw4200 with CPU of Intel(R) Pentium (R) 4, 3.8GHz, OS of Suse Linux version 9.2.
- The right-hand side $\boldsymbol{b}$ was imposed from physical conditions.
- The stopping criterion of the iterative methods is less than 10^{-6} of the relative residual 2-norm $||\boldsymbol{r}_{n+1}||_2/||\boldsymbol{r}_0||_2$.
- The maximum number of iterations is fixed as 10000.
- Computational times are written in seconds.
- Test matrices were derived from Florida sparse matrix collection.
- “TRR” shown in Tables denotes True Relative Residual of the converged solutions.

Test matrices

matrix	dim.	nnz	ave. nnz	anal. field
epb1	14,734	95,053	6.45	structural
epb2	25,228	175,027	6.94	
epb3	84,617	463,625	5.48	
poisson3Da	13,514	352,762	26.10	
poisson3Db	85,623	2,374,949	27.74	
xenon1	48,600	1,181,120	24.30	
*ex10hs	2,548	57,308	22.49	
raefsky2	3,242	294,276	90.77	hydro-
raefsky3	21,200	1,488,768	70.22	dynamic
*add20	2,395	17,319	7.23	circuit
*add32	4,960	23,884	4.82	circuit
memplus	17,758	126,150	7.10	electrical
wang3	26,064	177,168	6.80	
wang4	26,068	177,196	6.80	
k3plates	11,107	378,927	34.12	acoustics

Results for matrix poisson3Da

matrix	method	γ_k	$\boldsymbol{p}$	ω	itr.	time	$\log_{10}$(TRR)
poisson-3Da	GS	-	-	-	2148	7.15	-6.00
	IGS	1	$\boldsymbol{r}_0$	-	191	0.43	-6.44
			rand	-	174	0.39	-6.37
			con.	-	237	0.53	-6.32
	SOR	-	-	1.9	134	0.47	-6.04
	ISOR	1	$\boldsymbol{r}_0$	1.8	74	0.16	-6.01
			rand	1.8	76	0.18	-6.08
			con.	1.8	74	0.17	-6.01
		2	-	1.9	124	0.28	-6.02
	BiCGStab	-	(rand)	-	74	0.27	-6.02
	GPBiCG	-	(rand)	-	70	0.28	-6.04
	BiCGSafe	-	(con.)	-	70	0.28	-6.05

Results for matrix poisson3Db

matrix	method	γ_k	$\boldsymbol{p}$	ω	itr.	time	$\log_{10}$(TRR)
poisson-3Db	GS	-	-	-	break	-	-
	IGS	1	$\boldsymbol{r}_0$	-	819	15.49	-6.97
			rand	-	674	12.73	-6.66
			con.	-	744	14.06	-6.98
	SOR	-	-	all	break	-	-
	ISOR	1	$\boldsymbol{r}_0$	1.8	239	4.50	-6.03
			rand	1.8	249	4.69	-6.18
			con.	1.8	253	4.78	-6.11
		2	-	1.1	514	9.53	-6.00
	BiCGStab	-	(rand)	-	193	9.25	-6.02
	GPBiCG	-	(all)	-	-	-	-
	BiCGSafe	-	(con.)	-	175	7.19	-6.00

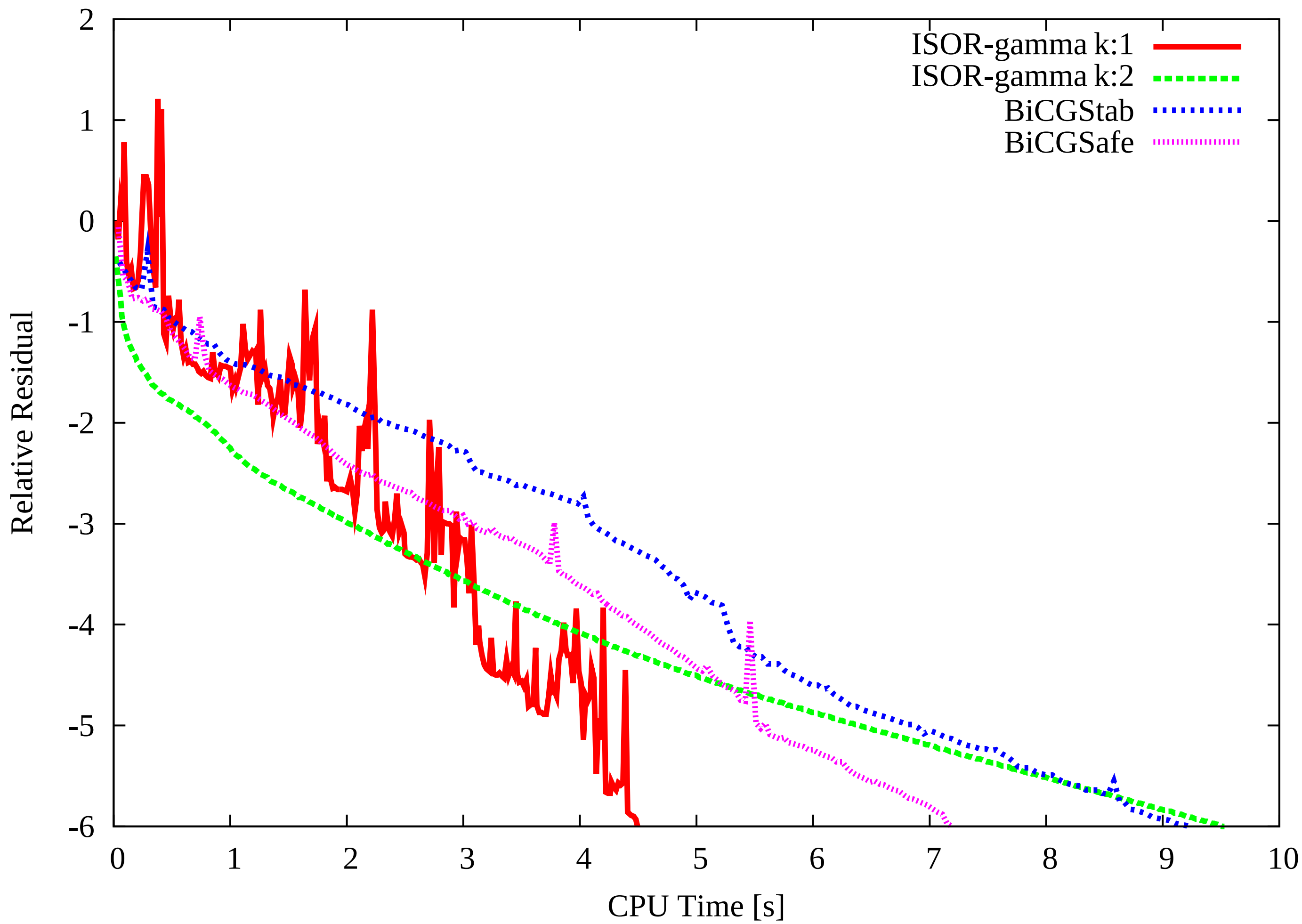
Results for matrix raefsky2

matrix	method	γ_k	$\boldsymbol{p}$	ω	itr.	time	$\log_{10}$(TRR)
raef-sky2	GS	-	-	-	max	19.74	111.36
	IGS	1	$\boldsymbol{r}_0$	-	308	0.35	-5.78
			rand	-	321	0.36	-5.70
			con.	-	329	0.38	-5.91
	SOR	-	-	1.0	max	19.74	111.36
	ISOR	1	$\boldsymbol{r}_0$	1.0	339	0.40	-6.16
			rand	1.0	353	0.41	-6.02
			con.	1.0	349	0.42	-6.06
		2	-	1.2	4101	4.73	-6.00
	BiCGStab	-	(con.)	-	304	0.65	-6.02
	GPBiCG	-	(rand)	-	289	0.64	-6.15
	BiCGSafe	-	($\boldsymbol{r}_0$)	-	291	0.63	-6.17

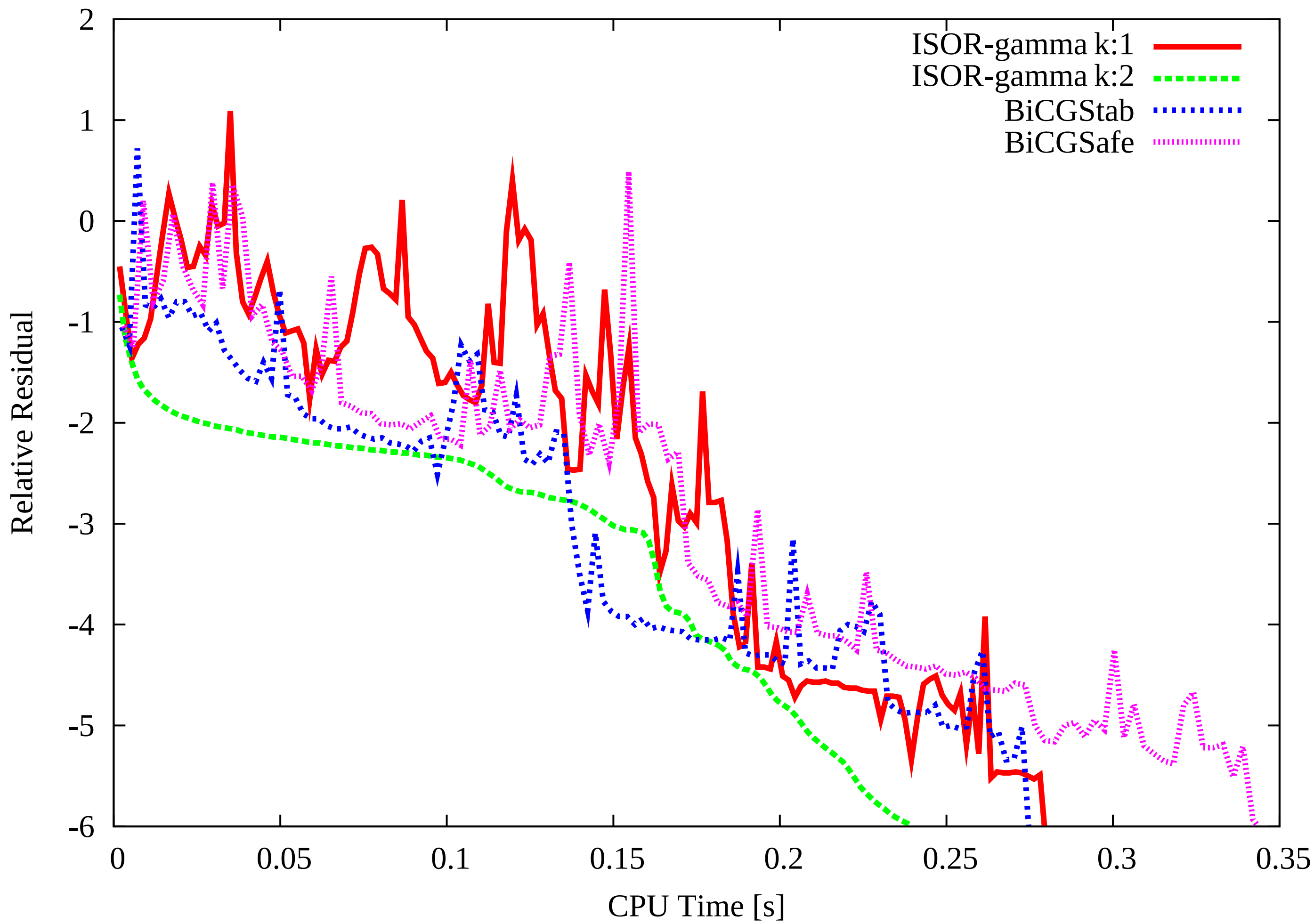
Results for matrix raefsky3

matrix	method	γ_k	$\boldsymbol{p}$	ω	itr.	time	$\log_{10}$(TRR)
raef-sky3	GS	-	-	-	break	-	-
	IGS	1	$\boldsymbol{r}_0$	-	1832	9.81	-6.08
			rand	-	1979	10.52	-6.09
			con.	-	1804	9.59	-6.12
	SOR	-	-	all	break	-	-
	ISOR	1	$\boldsymbol{r}_0$	1.0	1915	10.77	-6.13
			rand	1.0	1907	10.71	-6.05
			con.	1.0	1899	10.65	-6.01
		2	-	1.0	max	55.69	65.73
	BiCGStab	-	(rand)	-	1435	15.00	-6.00
	GPBiCG	-	(rand)	-	1395	15.92	-6.00
	BiCGSafe	-	(rand)	-	1407	15.61	-6.01

History of residual for poisson3Db



History of residual for epb2



Concluding remarks

- We proposed IDR-based Jacobi, Gauss-Seidel, SOR and ISOR(s) methods.
- We verified efficiency, robustness and stability of the proposed IDR-based iterative methods.
- As a result, it turned out that IJacobi, IGS and ISOR methods outperform compared with the conventional iterative methods.