**課題名（タイトル）：**

# 光格子における超流動 Fermi 気体のバンド構造

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1.　本課題の研究の背景、目的、関係するプロジェクトとの関係

Ultracold gases in optical lattices provide a new frontier of research where many remarkable phenomena can be observed and investigated. By using Feshbach resonances one can tune the interaction between atoms and investigate the crossover between the BCS state and a Bose-Einstein condensate (BEC), passing through a resonant regime where the scattering length is very large and the system exhibits universal properties (unitary regime).

Interestingly, in the case of BECs in one-dimensional (1D) optical lattices, the energy band shows loop structures (called the "swallow-tail" structure) due to the nonlinear effects provided the interaction is strong enough compared to the lattice potential. Such loop structures result in, e.g., a breakdown of the Bloch oscillation, etc. On the other hand, for Fermi superfluids, little has been studied so far and existence of swallow tails has been still unclear. To solve this problem, we study the band structure of Fermi superfluids in the 1D periodic potential in the BCS-BEC crossover. Main purpose of this project is to explore whether there exist such loop structures in Fermi superfluids, especially focusing on the most intriguing unitary regime in the BCS-BEC crossover. If we find the existence of the swallow-tail band structure in Fermi superfluids, we shall solve the following issues.

1) Difference and similarity between Fermi superfluids and BECs. We shall pursue unique properties of swallow tails of Fermi superfluids which are absent for BECs.

2) Experimental consequences of swallow tails in the BCS-BEC crossover.

3) A criterion for appearance of the swallow tails in the BCS and unitary regimes.

2.　具体的な利用内容、計算方法

In this project, in order to investigate the band structure of ultracold superfluid Fermi gases, we use a mean-field theory based on the Bogoliubov - de Gennes (BdG) equations. We study the whole region along the BCS-BEC crossover including the unitary regime at zero temperature focusing on the situation in which the lattice potential is relatively weak as in the recent experiments. In such a situation, the tight binding description is not adequate and a full numerical approach based on the BdG equations is called for. Although approximate, this approach captures basic features along the whole BCS-BEC crossover, including the formation of molecules and the most challenging unitary limit where, for uniform 3D configurations, the predictions are in reasonably good agreement with ab initio Monte Carlo simulations. The BdG equations apply also to situations where the density varies over distances of the order of the healing length.

3.　結果

We have been investigating a wide range of parameters including deep BCS and BEC regimes to obtain a systematic understanding. Results which we have obtained so far are as follows.

1) We have found that the energy band has a swallow-tail-like structure in the crossover region between the BCS and BEC limits.

2) Around unitarity, where the scattering length

diverges, the width of the swallow tail has a maximum. Such a non-monotonic behavior along the BCS-BEC crossover is very unique.
3) In the BEC region, the width of the swallow tail increases by approaching the unitarity because of an increase of the intermolecular interaction energy. On the other hand, in the BCS region, the increase of the width of the swallow tail is due to the enhancement of the paring field by approaching the unitary regime.
4) We have studied the critical velocities of the Landau instability along the BCS-BEC crossover. We have found that, when the lattice height is much larger than the Fermi energy, the periodic potential makes pairs of atoms to be strongly bound even in the BCS regime and pair-breaking excitations are suppressed.

4. まとめ

We have been studying the band structure of the ultracold superfulid Fermi gases in one-dimensional periodic potentials. Using the numerical simulation based on the BdG equations, we have established that the swallow tail exists also in the Fermi superfluids due to the non-linearity by the gap parameter in the BdG equations. We have also found that the width of the swallow tail shows a maximum in the unitary regime between the BCS and BEC limits.

5. 今後の計画・展望

We need to further study various parameters systematically to obtain a complete understanding of this phenomena. We are now performing such numerical calculations using RICC. We would like to find unique properties of swallow tails in Fermi superfluids which cannot be obtained in the BEC case. We also want to understand the behavior of the width of the swallow tail analytically based on the knowledge obtained from our numerical calculations.

6. RICC の継続利用を希望の場合は、これまで利用した状況（どの程度研究が進んだか、研究においてどこまで計算出来て、何が出来ていないか）や、継続して利用する際に行う具体的な内容

We are now trying to find unique properties of swallow tails in Fermi superfluids. For this purpose, we will perform further numerical calculations for various sets of parameters, especially deep BEC regime to see differences in swallow tails between BCS and BEC regimes. At the present moment, a simple and analytical understanding (i.e., a criterion for appearance of the swallow tail, etc.) is also lacking. We will pursue this issue with the help of the knowledge obtained by the numerical calculations.

7. 一般利用で演算時間を使い切れなかった理由

8. 利用研究成果が無かった場合の理由

**平成 22 年度　RICC 利用研究成果リスト**

**【論文、学会報告・雑誌などの論文発表】**

G. Watanabe, F. Dalfovo, L. P. Pitaevskii, and S. Stringari:
"Effects of periodic potentials on the critical velocity of superfluid Fermi gases in the BCS-BEC crossover"
to appear in Phys. Rev. A (arXiv:1010.4852 [cond-mat. quant-gas]).

**【国際会議などの予稿集、proceeding】**

Gentaro Watanabe:
"Simulating Pasta Phases by Molecular Dynamics and Cold Atoms — Formation in Supernovae and Superfluid Neutrons in Neutron Stars"
Proceedings of the Mini Symposium on Nuclear Astrophysics in "New Frontiers in QCD 2010"
Prog. Theor. Phys. Suppl. **186**, 45 (2010) [invited].

**【国際会議、学会などでの口頭発表】**

Gentaro Watanabe:
"Critical velocity of superflow through single-barrier and periodic potentials"
Fifth International Workshop on "Theory of Quantum Gases and Quantum Coherence"
(University of Nice, Sophia Antipolis, Nice, 2 June 2010) [invited].

Gentaro Watanabe:
"Superfluid unitary Fermi gases in a 1D optical lattice — thermodynamic properties and critical velocity"
Workshop on "Quantum solids, liquids, and gases"
(NORDITA, Stockholm, 2 August 2010) [invited].

Gentaro Watanabe:
"Properties of superfluid Fermi gases in a 1D optical lattice"
Advanced Program on "Quantum Condensation"
(National Tsin-hua University, Hsinchu, Taiwan, 13 August 2010).

**【その他】**